Chapter 2
Analytical Modeling of Concrete Hydraulic Structures

2-1. General

Structural modeling procedures consistent with the time-history method of analysis are discussed in this chapter. First the massive concrete hydraulic structures are defined and their general dynamic characteristics described. Then modeling procedures applicable to each hydraulic structure and fluid-structure and foundation-structure interactions are discussed, followed by description and application of the earthquake input motions required in the analysis. In general, the structural models developed should consider the most important dynamic characteristics of each particular hydraulic structure, including the fluid-structure and foundation-structure interactions. The effects of dynamic backfill pressures on the navigation lock walls and the reservoir boundary absorption for concrete dams can also be significant and should be considered (1-7*f*(4)). The structural modeling of a particular hydraulic structure is generally accomplished using a beam and/or finite element idealization of the structure. However, modeling for the fluid-structure interaction, foundation rock-structure interaction, and SPSI is more involved and may include simple procedures as well as more elaborate formulations. Earthquake input motions are described, and the manner in which they are applied to the structure is also discussed.

2-2. Types of Concrete Hydraulic Structures

Hydraulic structures considered in this manual include massive concrete structures designed for impounding water, regulating the release of impounded water, or fulfilling navigation demands during the periods when the riverflows are inadequate. These massive concrete structures include gravity and arch dams, intake towers, U-frame and W-frame navigation locks, and their associated approach walls.

2-3. Concrete Gravity Dams

a. Gravity dams are massive concrete hydraulic structures that retain the impounded water by resisting the forces imposed on them mainly by their own weight (Figure 2-1a). They are designed so that every unit of length is stable independent of the adjacent units. Although a typical gravity dam is usually straight, the dams are sometimes curved in plan to accommodate site topography and to gain added stability through arch action. The construction materials for gravity dams have evolved from stone masonry of the historic dams to mass concrete. To reduce the construction costs in comparison with those of the earth-fill and rock-fill dams, more gravity dams are now being built using the roller-compacted concrete (RCC), which uses earth-fill construction techniques.

b. Traditionally, analysis of a gravity dam considered a very simple mathematical model of the structure. Such a method was based on the concept that the resistance to external forces was 2-D in nature, so only a unit slice of the dam taken in the upstream-downstream direction was analyzed. The earthquake forces were expressed as the product of a seismic coefficient and were treated simply as static forces. Only the effects of horizontal ground motion applied in the upstream-downstream direction were considered. However, to represent the resistance mechanism realistically, it now has become standard practice to use some form of

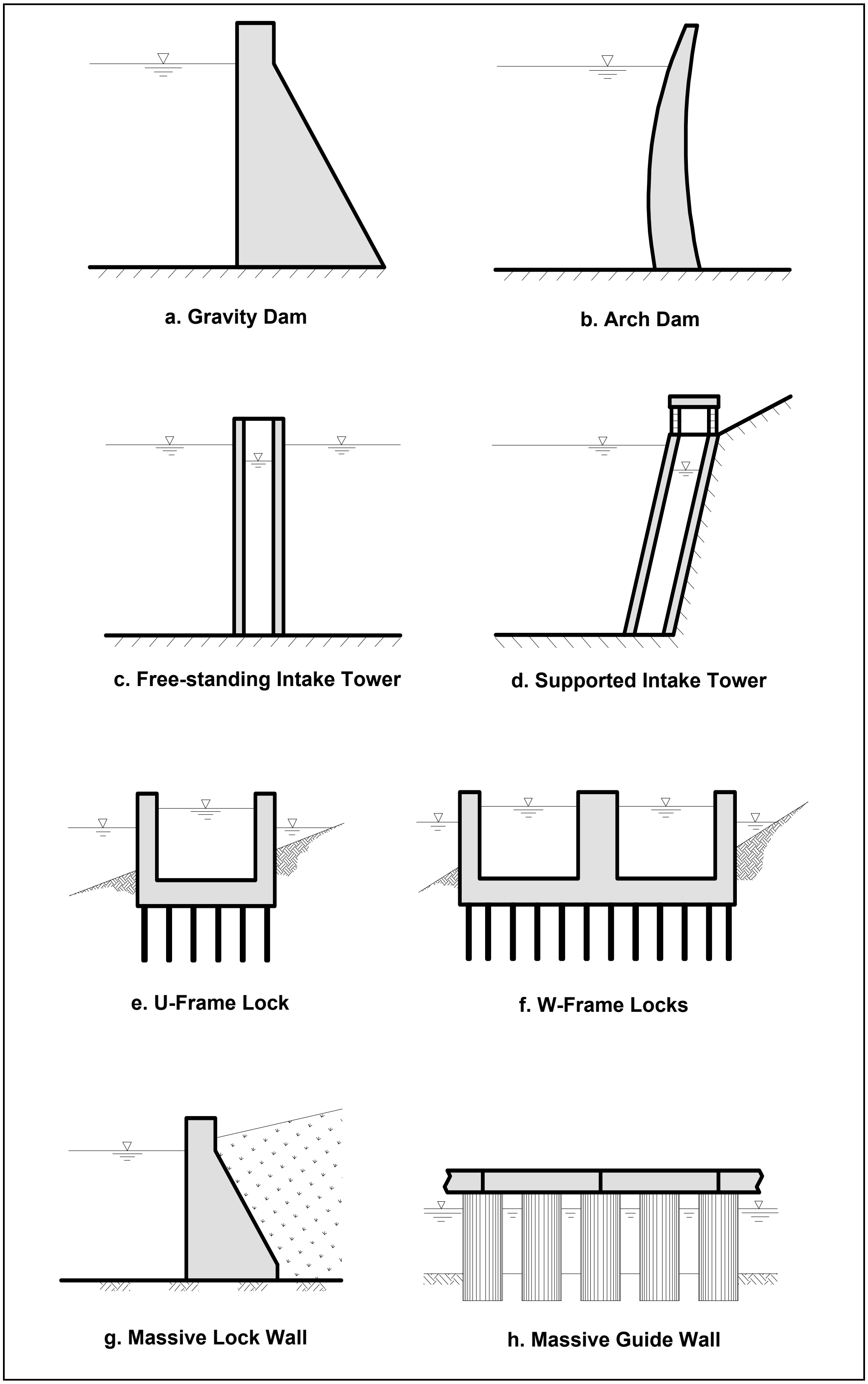


Figure 2-1. Types of concrete hydraulic structures

finite element model in the analysis of both the static and dynamic response of concrete gravity dams. The finite element method not only represents the resistance mechanism more realistically, but it also takes into account the dynamic characteristics of the dam-water-foundation system as well as the characteristics of earthquake ground motion. The inherent strength and stiffness of gravity dams in the cross-stream direction are so great that there usually is no concern about their response to earthquake excitation acting in that direction. However, it has been demonstrated (Chopra 1987) that the dynamic response of a gravity dam to the vertical component of the earthquake motions may be similar in amplitude to that induced by the horizontal ground motions and should be considered in the analysis.

2-4. Concrete Arch Dams

a. An arch dam (Figure 2-1b) is a solid concrete hydraulic structure curved in plan and possibly in elevation, which transmits a large portion of the water pressure and other loads by means of thrust (arch action) to the abutment, therefore utilizing the compressive strength of its material. Deriving their structural behavior from material strength rather than the sheer weight of the dam, arch dams are recognized for their structural competence and economical construction.

b. Arch dams are generally classified as thin, medium-thick, and thick arch dams. Their shapes have gradually changed from the early circular, horizontal, constant-thickness arches and vertical upstream faces to the more elaborate thin double-curvature arch dams with variable radii, variable thickness, and noticeable overhangs.

c. The response of an arch dam to ground shaking is similar to that of a gravity dam in the sense that both types are subject to dynamic response amplification in accordance with the relationships between the frequencies of the dam system and those of the earthquake motions. However, because of the complicated 3-D configuration of an arch dam, its response must be considered with regard to all three components of seismic input. Hence a rather refined 3-D model must be formulated of the dam and its foundation to calculate the vibration properties that control the dynamic amplification of the input accelerations. The inertial and damping effects of the reservoir water have an important effect on the vibration properties and response of an arch dam as they do for a gravity dam, and thus they must be included in the mathematical model as described in paragraph 2-13*c*.

2-5. Intake-outlet Towers

a. Intake-outlet towers form the entrance to reservoir outlet works. They are often equipped with gates for regulating the release of water, or for lowering the reservoir as a precautionary measure after a major earthquake event. The failure of an intake-outlet tower during an earthquake could possibly the disrupt delivery of important public services and sometimes contributed to failure of a dam. It is therefore important that intake-outlet towers in seismically active regions be designed or evaluated to withstand earthquakes using rational analytical methods, which are based on a sound understanding of the dynamic behavior of the tower-water-foundation system.

b. Most intake-outlet towers are free-standing structures surrounded by water and founded on an enlarged base on the reservoir bottom or partially encased through bedrock or hard soil (Figure 2-1c). Some are embedded within embankment dams; others are structurally connected to the upstream of concrete dams. There are yet other intake towers that are inclined against the rock slopes and partially embedded into the bedrock formation (Figure 2-1d). The intake structures at Seven Oaks Dam in southern California and at Cerrillos Dam in Puerto Rico are two examples of inclined towers designed by USACE. The design of Seven

Oaks Dam intake tower also included anchoring it to the inclined slope rock to provide resistance for severe earthquake force demands of the southern California earthquakes.

c. Regardless of their types, intake towers are surrounded by water, sometimes to a significant height, and may contain internal water. Intake towers are therefore subjected to the fluid-structure interaction effects that can significantly influence their responses to earthquakes. The response of intake towers is also influenced by soil-structure interaction (embankment, rock, or soil foundation), and possibly by the access bridge, mass of the internal equipment, and response of the dam when the towers are tied to concrete dams. Depending on the complexity of the geometry of the tower, stick models with beam element and lumped masses or 3-D finite element mathematical models are required to adequately represent their vibration characteristics and their dynamic responses.

2-6. U-frame and W-frame Navigation Locks

Navigation locks are massive concrete hydraulic structures designed to provide navigable pass for towboats during the periods when the riverflows are inadequate. A U-frame (single chamber, Figure 2-1e) or W-frame (dual chamber, Figure 2-1f) lock consists of reinforced concrete walls constructed integrally with a reinforced floor slab founded on a pile foundation. On major navigable rivers, locks are usually built in conjunction with navigation dams, and may cover several hundreds to over one thousand feet in total length. A typical lock normally includes numerous monoliths of variable length and type separated by construction joints. The chamber monoliths make up the main portion of the lock, while the culvert intake, upper gate, culvert valve, culvert discharge, and lower gate monoliths serve as the special-purpose monoliths covering the remaining length of the structure. Lock structures contain water and are bounded by water on the river wall, which may reach a significant height. Thus they are subjected to fluid-structure interaction effects that can influence their response to earthquakes. The response of lock structures can also be affected significantly by the interactions with the soil-pile foundation and the backfill soil. Considering that the SPSI also affects the seismic input motion, SPSI analyses are required to adequately represent both the seismic input and the dynamic response of the pile-founded locks.

2-7. Massive Concrete Lock Walls

At locations where sound and durable rock is available, lock chambers and their extensions upstream and downstream may be constructed with separate massive walls founded on the foundation rock (Figure 2-1g). These gravity-type lock walls usually have a thin or nonexistent floor section. The land wall and river wall may therefore be treated as separate gravity sections subjected to all applicable loads discussed in paragraph 2-16.

2-8. Massive Concrete Guide Walls

a. Massive concrete guide walls control navigation conditions in the upper and lower approach areas of a lock structure. The purpose of the walls is to guide towboat traffic and other vessels into and out of the locks. Guide walls are designed as either a drilled shaft or cellular fixed wall or a floating pontoon wall. Fixed guide walls (Figure 2-1h) normally consist of cast-in-place concrete walls supported on circular sheet pile cells. The loads applied to the foundation from the fixed walls are usually high due to the large quantity of concrete resting on the cells. High seismic loads, large towboat impact loads, and the dead load from the fixed guide walls may require that the cells be founded on steel piles driven through the cells and into the competent rock formation.

b. The floating guide walls usually consist of a floating pontoon spanning pylons, one at each end. The wall may also include a nose pier at one end for protection against direct impacts. A single drilled shaft several feet in diameter may support the pylons while three or more drilled shafts may be required to support the nose piers. The drilled shafts normally consist of a steel pipe filled with reinforced concrete. Similar to the fixed walls, floating approach walls are also designed to resist a variety of loads including barge impact, wind, current forces, and seismic base motions. Both the fixed and floating walls are subjected to the fluid-structure interaction effects that can significantly influence their responses to earthquake and impact loading. The response of approach walls is also influenced by the SPSI effects and should be represented adequately in the analysis.

2-9. Analytical Modeling Procedure

Modeling procedures for time-history analysis of hydraulic structures including the effects of fluid-structure interaction and foundation-structure interaction can be classified into the standard finite element and the substructure methods. In the standard finite element method, the complete structure-water-foundation system is modeled as a single or composite unit. The substructure method consists of dividing the complete system into three substructures: the structure, the water, and the foundation, each of which can be partly analyzed independently of the others. The main difference between the two methods in terms of structural modeling is that the interaction effects of the water and foundation are more accurately represented by the substructure method.

2-10. Substructure Method

a. As stated previously, in the substructure method, the complete system is divided into three substructures: the structure, the water, and the foundation rock region (Figure 2-2). The structure is normally represented as a beam or 2-D or 3-D finite element system, which permits modeling of a general geometry and linear elastic material properties. The water domain may be idealized as a continuum or as a combination of a finite element and continuum system. Dynamic interaction between the structure and the water is expressed as frequency-dependent or frequency-independent hydrodynamic forces at the structure-water interface. The foundation region may also be idealized as a continuum (Dasgupta and Chopra 1979) or as a finite element system. The continuum idealization permits accurate modeling of the structure-foundation interaction when similar materials extend to large depths. For sites where soft rock or soil overlies harder rock at shallow depths, a finite element idealization of the foundation region is more appropriate. Dynamic interaction between the structure and the foundation is expressed by interacting forces at the structure-foundation interface (i.e., base nodes). These interacting forces are frequency-dependent and are related to the displacements through the dynamic stiffness (impedance) matrix for the foundation region.

b. The equations of motion for the structure including the effects of the structure-water interaction and the structure-foundation interaction most conveniently are expressed in the frequency domain, because the hydrodynamic forces and the impedance functions for the foundation region depend on the frequency of excitation. The general form of the frequency domain equations of motion for a structure-water-foundation system is given by

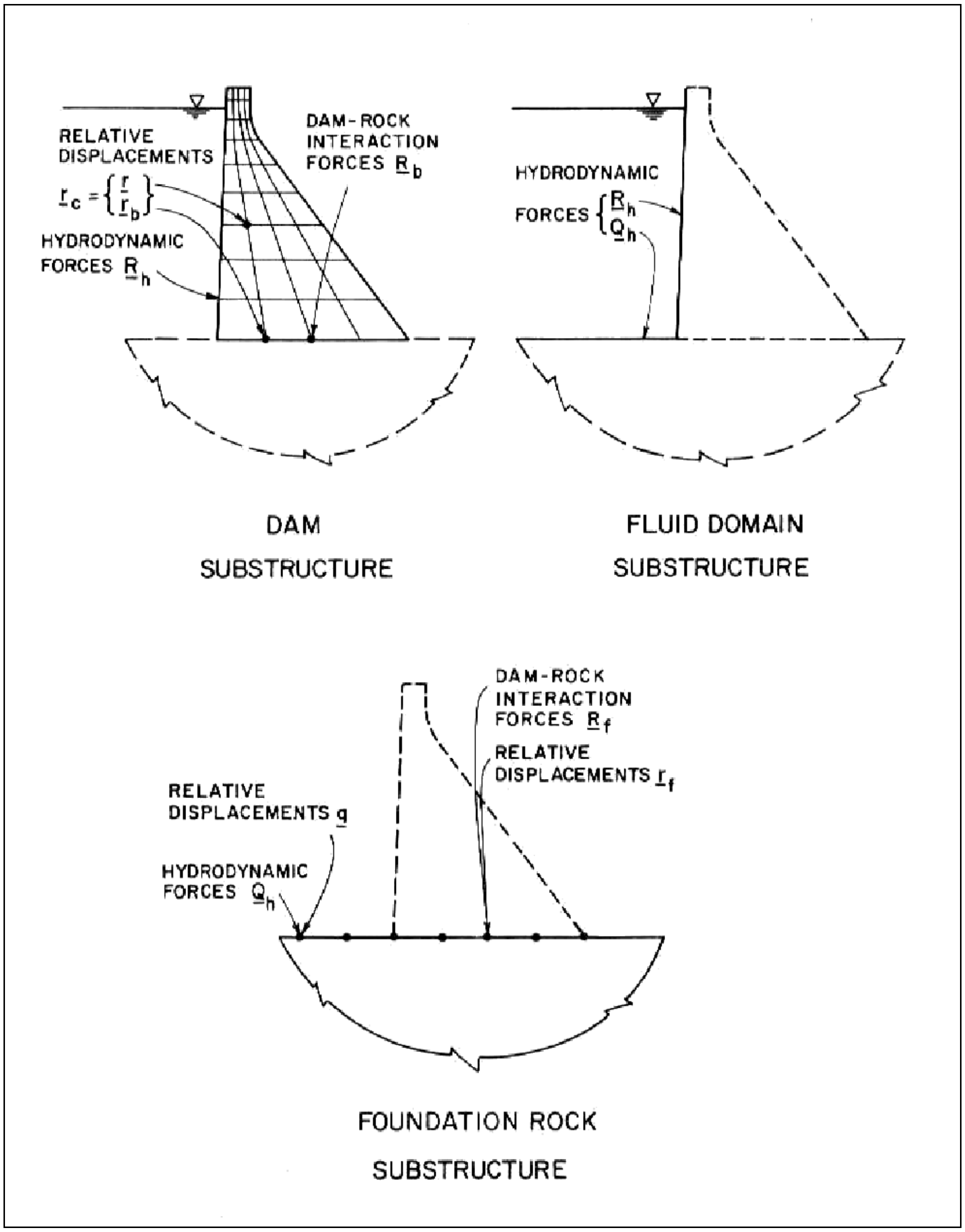


Figure 2-2. Substructure representation of dam-water-foundation system (from Fenves and Chopra 1984b)

$$\left\{-\omega^2 \begin{bmatrix} m & \underline{0} \\ \underline{0} & m_b \end{bmatrix} + (1+i\eta_s)\begin{bmatrix} k & k_b \\ k_b^T & k_{bb}\end{bmatrix} + \begin{bmatrix}\underline{0} & \underline{0}\\ \underline{0} & \underline{S}_f(\omega)\end{bmatrix}\right\}\begin{Bmatrix}\bar{r}^l(\omega)\\ \bar{r}_b^l(\omega)\end{Bmatrix} = -\begin{Bmatrix} m\underline{1}^l \\ m_b \underline{1}_b^l\end{Bmatrix} + \begin{Bmatrix}\bar{R}_h^l(\omega)\\ -\underline{S}_{rq}\underline{S}_{qq}^{-1}\bar{Q}_h(\omega)\end{Bmatrix} \tag{2-1}$$

where

ω = harmonic excitation frequency
m = mass submatrix corresponding to nodal points above the base
m_b = mass submatrix corresponding to nodal points at the base
i = $\sqrt{-1}$
η_s = constant hysteretic damping factor for the structure
k = stiffness submatrix corresponding to nodal points above the base
k_{bb} = stiffness submatrix corresponding to nodal points at the base
k_b = coupling stiffness submatrix relating nodal points above the base to nodal points at the base
k_b^T = transpose of k_b
$S_f(\omega)$ = dynamic stiffness matrix of the foundation region defined with respect to nodal points at the base
$\bar{r}^l(\omega)$ = frequency response functions for nodal displacements above the base ($l = x, y$, or z) (see Figure 2-2)
$\bar{r}_b^l(\omega)$ = frequency response function for nodal displacements at the base (see Figure 2-2)
$\underline{1}^l$ = subvector of 1's corresponding to nodal points above the base (see Figure 2-2)
$\underline{1}_b^l$ = subvector of 1's corresponding to nodal points at the base
$\overline{R}_h^l(\omega)$ = vector of frequency response functions for hydrodynamic forces corresponding to nodal points at the structure-water interface (see Figure 2-2)
$\underline{S}_{rq}$ = coupling submatrix of dynamic stiffness of foundation region relating nodal points at the foundation surface under the structure base to nodal points at the foundation surface beneath the water
$\underline{S}_{qq}$ = Submatrix of dynamic stiffness of foundation region defined with respect to nodal points at the foundation surface beneath the water
$\overline{Q}_h(\omega)$ = vector of frequency response functions for hydrodynamic forces corresponding to nodal points at the foundation surface beneath the water (see Figure 2-2)

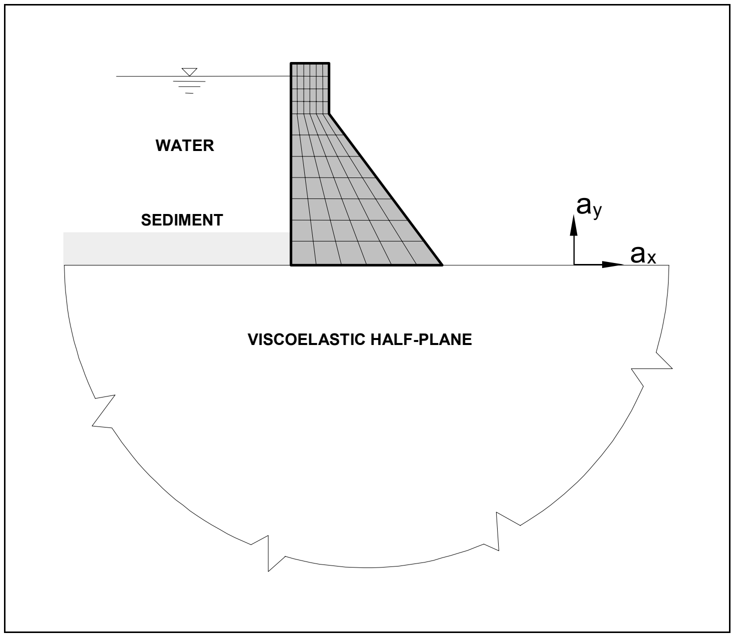
The foundation impedance matrix relates the interacting forces and displacements relative to the free-field ground motion in the l^{th} direction as given by

$$S_f(\omega)\bar{r}_f^l(\omega) = \overline{R}_f^l(\omega) \tag{2-2}$$

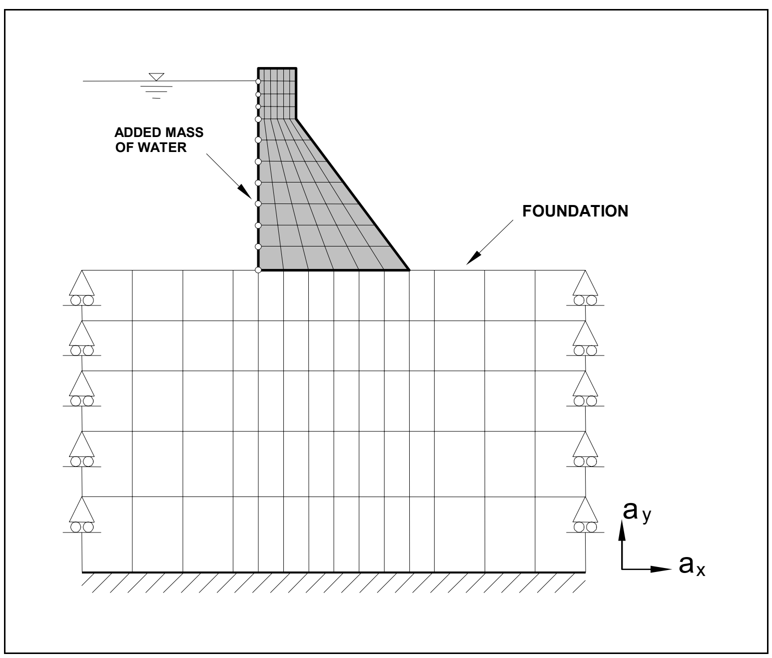
The hydrodynamic forces and the foundation impedance functions are obtained from the separate analysis of the fluid domain and the foundation rock substructures.

2-11. Standard Finite Element Method

In the standard finite element approach, the complete system consisting of the structure, the water, and the foundation region is modeled and analyzed as a single composite structural system . Similar to the substructure approach, the structure is modeled as an assemblage of beams or finite elements. The water and the foundation are generally represented by simplified models that only approximately account for their interactions with the structure. In most cases the water is modeled by an equivalent added hydrodynamic mass, and the foundation rock region is represented by a finite element system accounting for the flexibility of the foundation only. Based on these assumptions the equations of motion for the complete system become



a. Substructure model



b. Standard model

Figure 2-3. Finite element model of gravity dam

$$(m_s + m_a)\ddot{r} + c\dot{r} + kr = -(m_s + m_a)\underline{1}_s^x a_g^x(t) - (m_s + m_a)\underline{1}_s^y a_g^y(t) - (m_s + m_a)\underline{1}_s^z a_g^z(t) \qquad (2\text{-}3)$$

where

m_s = mass matrix of the structure
m_a = added hydrodynamic mass matrix having nonzero terms only at the structure-water nodal points
$\dot{r}, \ddot{r}$ = velocity and acceleration vectors, respectively
c = overall damping matrix for the entire system
k = combined stiffness matrix for structure and foundation region
r = vector of nodal point displacements for the complete system relative to the rigid base displacement
$\underline{1}_s^x$ = vector of 1's corresponding to x-DOFs
$a_g^x(t)$ = ground acceleration input in x-direction

The added hydrodynamic mass generally includes nonzero terms for x-, y- and z-DOFs, because they arise from the hydrodynamic pressures acting normal to the structure-water interface. For the structure-water interface with simple geometry, the added hydrodynamic mass terms associated with certain DOFs may be zero. For example, only the added hydrodynamic mass terms corresponding to the x-DOFs (horizontal direction) are nonzero for a gravity dam having vertical upstream face.

2-12. Concrete Gravity Dams

Conventional concrete gravity dams are constructed as monoliths (blocks) separated by transverse contraction joints. Oriented normal to the dam axis, these vertical joints extend from the foundation to the top of the dam and from the upstream face to the downstream face. For the amplitude of motion expected during strong earthquakes, the shear forces transmitted through the contraction joints are small compared with the inertia forces of the monoliths. For this condition, the monoliths in a long and straight gravity dam tend to vibrate independently, and their responses to earthquakes can be evaluated on the basis of a 2-D model. However, curved gravity dams and those built in narrow canyons need to be analyzed using a 3-D model.

a. 2-D gravity dam model. A 2-D model of a gravity dam for the time-history earthquake analysis consists of a monolith section supported on the flexible foundation rock and impounding a reservoir of water. The tallest monolith or dam cross section is usually selected and modeled using plane stress finite elements. The 2-D model of the selected monolith and the associated foundation rock and the impounded water may be developed as separate systems using the substructure method (Figure 2-3a), or as a complete structural system employing the standard finite element procedures (Figure 2-3b).

(1) Substructure method. In the substructure method the foundation is usually modeled as a viscoelastic half plane (paragraph 2-24*b*(1)). The interaction between the dam and foundation is represented by an impedance matrix (i.e., $S_f(\omega)$ in Equation 2-2), defined with respect to the nodal points at the dam-foundation rock interface. The impedance matrix for the viscoelastic half plane is obtained from a separate continuum solution of the foundation region (Dasgupta and Chopra 1979). Assuming that reservoir water can be modeled as a fluid domain with constant depth and infinite length in the upstream direction, the frequency-dependent hydrodynamic forces at the dam-water interface also are obtained from a separate continuum solution (paragraph 2-21). The seismic input includes the vertical and one horizontal component of the free-field acceleration time-histories applied at the dam-foundation and at the water-foundation interface regions. Since the foundation impedance matrix and hydrodynamic forces are frequency dependent, the dam response is first carried out in the frequency domain using a discrete Fourier transformation and then the results are presented in the time domain by the application of an inverse discrete Fourier transformation (Fenves and Chopra 1984b).

(2) Standard method. The viscoelastic half plane model discussed in (1) above is applicable to a homogeneous foundation where identical rock properties are assumed to exist for the entire unbounded foundation region. In general, foundation rock properties vary with depth and along the footprint of the dam. The effective modulus of the jointed rock within the shallow depths may significantly differ from that at greater depths. In these situations the viscoelastic half plane model is not appropriate and needs to be replaced by a finite element foundation model that can account for the variation of rock properties. The standard procedure is to develop a complete finite element model, which consists of the dam and an appropriate portion of the foundation region, as shown in Figure 2-3b. The foundation model, however, is assumed to be massless in order to simplify the application of the seismic input and avoid the use of large foundation models (paragraph 2-24a). The foundation mesh needs to be extended a distance at least equal to the dam height in the upstream, downstream, and downward directions. The nodal points at the base of the foundation mesh are fixed both in the vertical and horizontal directions. The side nodes, however, are attached to horizontal roller supports for the horizontal excitation and to vertical roller supports for the vertical excitation of the dam. The earthquake ground motions recorded at the ground surface are directly used as the seismic input and are applied at the base of the foundation model. The impounded water is also assumed to be incompressible so that the dam-water interaction effects can be represented by the equivalent added-mass concept. The added mass is obtained using either the simplified procedure developed by Fenves and Chopra (1986) or the generalized Westergaard method described in paragraph 2-19b.

b. 3-D gravity dam model. Sometimes monolith joints are keyed to interlock two adjacent blocks, or the dam is built in narrow canyons or is curved in plan to accommodate the site topography and to transfer part of the water load to the abutments. In these situations, the dam behaves as a 3-D structure and its response especially to earthquake loading should be evaluated using 3-D idealization similar to that described for arch dams in paragraph 2-13.

2-13. Concrete Arch Dams

Because concrete arch dams are 3-D structures, their responses to earthquake loading must be evaluated using a 3-D model. The 3-D model for an arch dam is developed using the finite element procedures and includes the concrete arch, the foundation rock, and the impounded water (Ghanaat 1993a, 1993b). The arch dam-water-foundation system may be analyzed using the substructure method or the standard finite element procedures. Both methods use the same mathematical model to represent the concrete arch, except that the substructure method permits more rigorous analysis of the dam-foundation and the dam-water interaction effects (Tan and Chopra 1995). The standard method employs a massless foundation rock with an incompressible finite element model for the impounded water (Ghanaat 1993a, 1993b). The substructure method considers not only the foundation flexibility but also the damping and inertial effects of the foundation rock, and also includes a reservoir water model that accounts for the effects of water compressibility and the reservoir boundary absorption.

a. Dam model. Concrete arch dams are usually idealized as an assemblage of finite elements, as shown in Figures 2-4 and 2-5. The finite element model of the dam should closely match the dam geometry and be suitable for application of the various loads and presentation of the stress results. To the extent possible, the finite element model of an arch dam should be developed using a regular mesh with elements being arranged on a grid of vertical and horizontal lines (Figure 2-4). This way the gravity loads can easily be applied to the individual cantilever units, and the stresses computed with respect to local axes of the element surfaces would directly relate to the familiar arch and cantilever stresses. The finite elements appropriate for modeling an arch dam include 3-D solid and shell elements available in the computer program GDAP (Ghanaat 1993a) or a general 3-D solid element with 8 to 21 nodes (Bathe and Wilson 1976). A thin or medium-thick arch dam can be modeled adequately using a single layer of shell elements through the dam thickness. A thick arch dam may

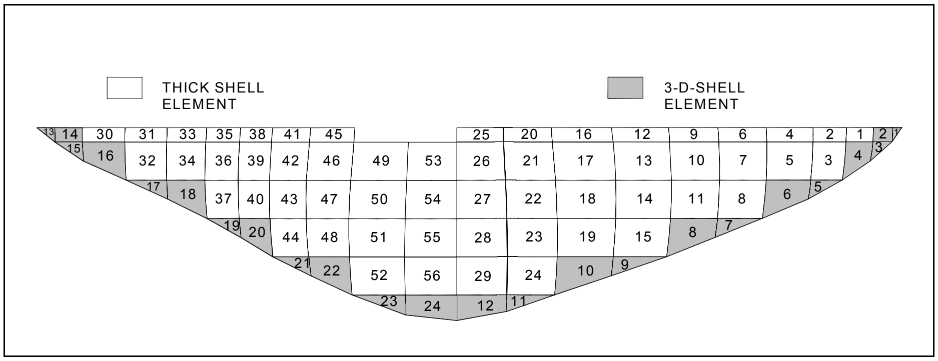


Figure 2-4. Finite element mesh of arch dam showing elements used in the dam

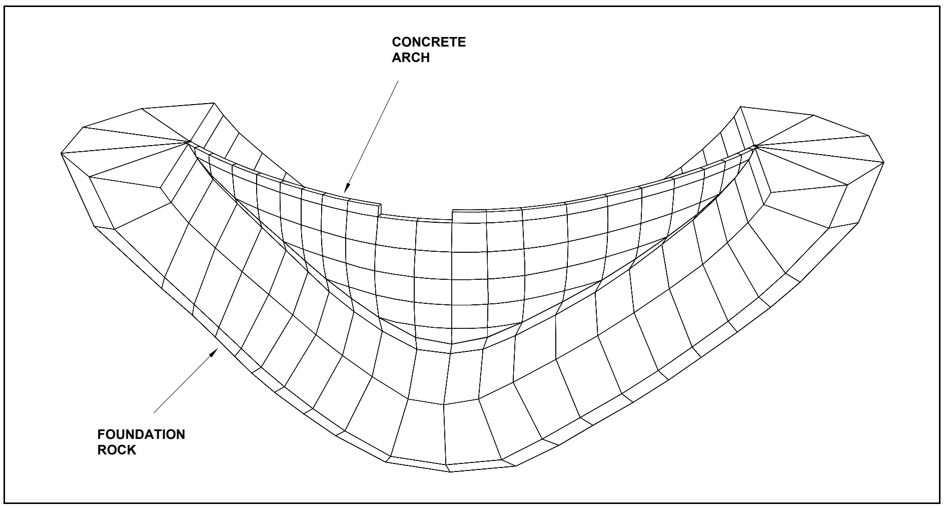


Figure 2-5. Perspective view of dam-foundation finite element model

require three or more layers of solid elements through the dam thickness to better represent its dynamic behavior. The level of finite element mesh refinement depends on the type of elements used. In general, a finite element mesh using the linear 8-node solid elements needs to be finer than that employing shell elements whose displacements and geometry are represented by quadratic functions.

b. Foundation model

(1) The standard foundation model for analysis of arch dams is the massless foundation discussed in paragraph 2-24*a*, in which only the effects of foundation flexibility are considered. Such a foundation model should extend to a distance beyond which its effects on deflections, stresses, and natural frequencies of the dam become negligible. The size of the foundation model should be determined based on the modulus ratio of the foundation to the concrete E_f/E_c. For a competent foundation rock with $E_f/E_c \geq 1$, a foundation mesh extending one dam height in the upstream, downstream, and downward directions is adequate. For a more flexible foundation rock with E_f/E_c in the range of 1/2 to 1/4, the foundation model should extend at least twice the dam height in all directions and include more elements. In general, the foundation model can be developed to match the natural topography of the foundation rock region. Such a refined model, however, is not usually required in practice. Instead, a prismatic model employed in the GDAP program (Ghanaat 1993a) and shown in Figure 2-5 may be used. The seismic input for the massless foundation model includes three-component ground acceleration time-histories applied at the fixed boundary nodes of the foundation mesh. Since no wave propagation takes place in the massless foundation model, the seismic input is obtained from

the earthquake motions recorded on the ground surface using scaling or spectrum-matching procedures described in Chapter 5.

(2) In the substructure method of arch dam analysis, the impedance matrix of the foundation rock region is employed to represent dam-foundation interaction effects (Tan and Chopra 1995). The impedance matrix (or frequency-dependent stiffness matrix) includes both the inertia and damping of the foundation rock region as well as its flexibility. This impedance matrix for arch dams is determined using a direct boundary element formulation applied to a uniform cross-section canyon cut in a homogeneous viscoelastic half space (Zhang and Chopra 1991). The assumption of a uniform cross-section canyon is to reduce the original 3-D boundary value problem into an infinite series of 2-D problems. The foundation model therefore is represented by the dam-foundation rock interface discretized into a set of boundary elements whose nodal points match the finite element idealization of the dam (Tan and Chopra 1995). The properties of the foundation rock are characterized by its Young's modulus, Poisson's ratio, and unit weight, which are assumed to be constant over the entire unbounded foundation region. Although this foundation model overcomes the limitations of the massless foundation, it overestimates damping for a foundation rock having relatively low modulus.

c. Reservoir water.

(1) The standard dam-water interaction analysis for arch dams is based on the finite element added hydrodynamic mass model described in paragraph 2-20a (Ghanaat 1993a). Assuming the water is incompressible, the hydrodynamic pressures acting on the dam-water interface are first obtained from the finite element solution of wave equation and then converted into equivalent added-mass terms. The resulting added-mass terms are subsequently combined with the mass of concrete nodal points on the dam-water interface. In most cases a prismatic finite element fluid mesh similar to that shown in Figure 2-22 (paragraph 2-20) is adequate for computation of the added hydrodynamic mass. However, for reservoirs with irregular topography and shape, a fluid mesh that matches the actual reservoir topography is recommended (Figure 2-6).

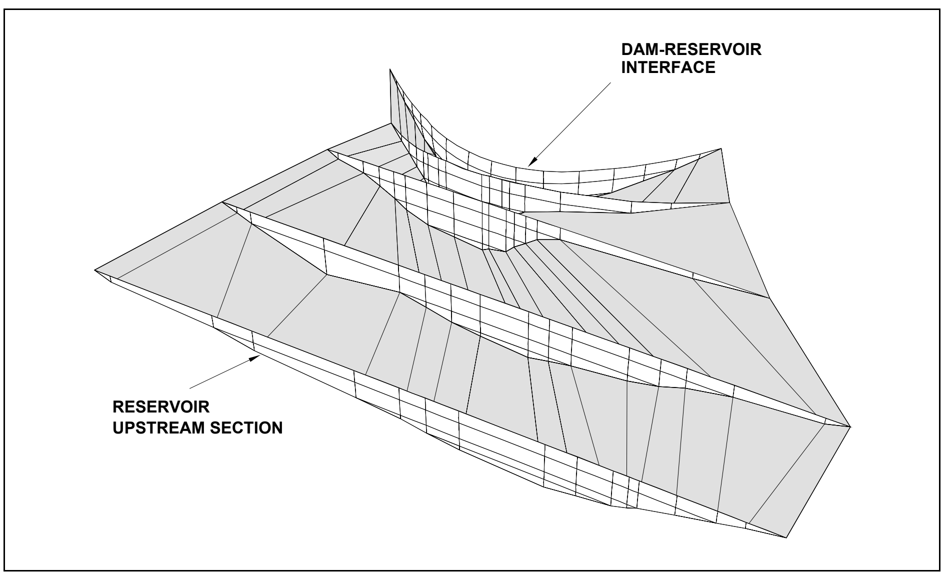


Figure 2-6. Finite element mesh for incompressible water developed to match reservoir bottom topography (shaded region)

(2) A rigorous analysis of the dam-water interaction may be required when the fundamental frequency of the reservoir water is relatively close to fundamental frequency of the dam. Such an analysis, which includes the effects of water compressibility and reservoir boundary absorption on the response of the dam, is performed as described in paragraph 2-21.

2-14. Intake-outlet Towers

Intake-outlet towers are designed in various structural configurations and geometric shapes. In terms of their structural configurations, they may be classified into two types: free-standing and supported towers. Free-standing towers are vertical structures typically founded on an enlarged base on the reservoir bottom or deeply embedded in bedrock or stiff soils. The elevation profile may be uniform or tapered, and the plan section may be rectangular, circular, or irregular. Supported towers are built against an abutment to provide increased structural stability and improved seismic performance. They may be designed as vertical or inclined towers supported in the lower portion or along the entire length. Like the free-standing towers, the supported towers are also designed in various geometric shapes, but they are subjected to earthquake excitation at the base as well as along the abutment supports. The following paragraphs discuss the modeling procedures for both types of towers.

a. Free-standing towers. Free-standing towers may be modeled using the substructure or standard finite element method of analysis. The available substructure method is restricted to a tower supported on the horizontal ground surface and has two axes of plan symmetry with homogeneous material properties for the unbounded foundation region (Goyal and Chopra 1989). The standard finite element model is not restricted to homogeneous material properties for the foundation but employs foundation models that account only approximately for the foundation-structure interaction. Each model is described in the following paragraphs.

(1) Standard finite element model.

(a) Free-standing towers with regular cross-section geometry, whose dimensions may remain constant or vary along the height of the tower, can be idealized adequately using beam elements as depicted in Figure 2-7. Slender towers exhibit primarily flexural behavior, and their responses may be approximated ignoring the shear deformations. The response of squat towers (i.e., towers with a height-to-width ratio less than 10), especially for higher modes, is affected by shear deformations and should be considered in the analysis by using beam elements that include shear deformation capabilities. The response of towers is also influenced by the water-structure interaction, foundation-structure interaction, mass of internal equipment, and possibly the access bridge. The water-structure interaction is approximated by the added hydrodynamic mass described in paragraph 2-19*d*. The added hydrodynamic mass and the mass of the structure are lumped at the element nodal points. The foundation-structure interaction effects are approximately represented by equivalent linear springs attached to the base or to the embedded part of the model. If permitted by the computer program selected for the analysis, a combination of springs and dashpots may be used to represent the unbounded foundation region (paragraph 2-25*a*). For towers accessed by a light footbridge, the effects of the bridge may be considered by simply lumping part or all of the mass of the bridge at the nearest nodal points. A massive reinforced concrete access bridge, however, may significantly influence the response of a flexible tower, and its effects should be accounted for by modeling the bridge as part of the tower structure, as shown in Figure 2-7. The effects of a flexible access bridge may be negligible on the response of a stiff tower because their fundamental frequencies are completely different, but the tower provides support and excitation to the bridge and its effects on the response of the bridge should be considered.

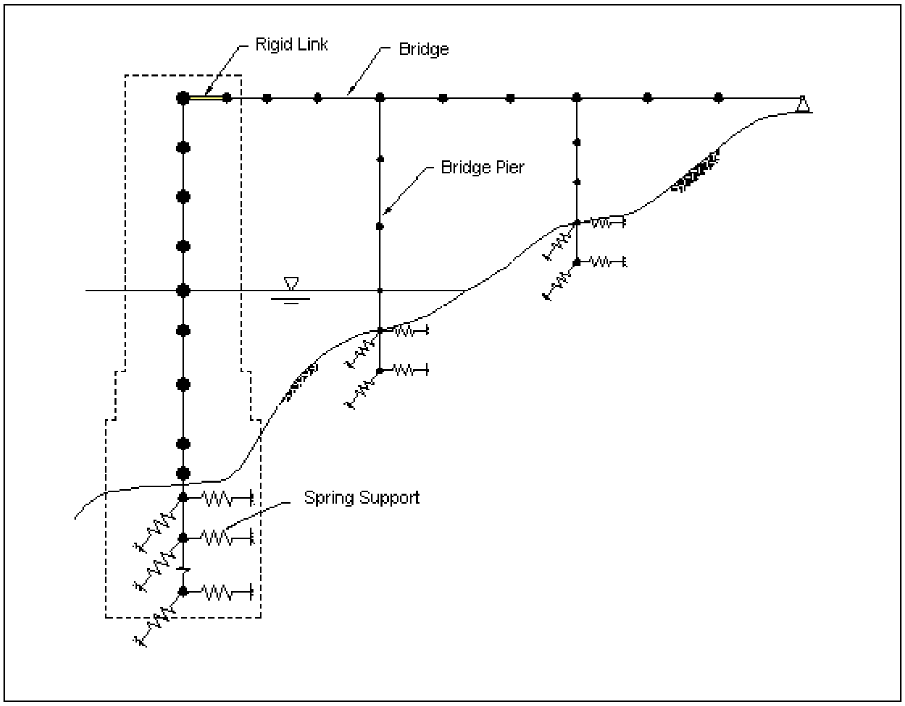


Figure 2-7. Finite element idealization of free-standing tower and access bridge

(b) The earthquake input for the tower beam model described in (a) above is defined by two horizontal components of the free-field ground acceleration time-histories. The effect of the vertical component of ground motion is expected to be negligible and is therefore not considered in the analysis. The ground motion is assumed to be identical at all support points.

(c) An irregular free-standing tower may require finite element discretization using solid elements as discussed in *b* below. In that case a finite element foundation model should also be developed. The earthquake excitation for such a model is defined by the vertical and two horizontal components of the free-field ground acceleration histories.

(2) Substructuring model.

(a) Free-standing towers supported on the horizontal surface of flexible rock or soil may be idealized as four separate substructures: the tower, the surrounding water domain, the inside water domain, and the foundation rock or soil system (Figure 2-8). The available substructuring procedure for intake-outlet towers assumes the tower to have arbitrary geometry, but it must have two axes of plan symmetry (Goyal and Chopra 1989). The tower is modeled as an assemblage of one-dimensional beam elements, including bending and shear deformations as well as rotatory interia (Figure 2-9). Each beam element node has two degrees of freedom, translational and rotational displacements. The part of foundation block above the ground surface may be assumed rigid or represented by beam elements as part of the tower; the remaining part below the ground surface is treated as a rigid footing of negligible thickness supported by a homogeneous viscoelastic half space. The impedance functions (frequency-dependent stiffness) of the homogeneous viscoelastic halfspace are obtained from the analytical solution available for a circular foundation. For noncircular foundations, the impedance functions are determined approximately by using an equivalent circular foundation that has the same area and moment of inertia as the actual foundation.

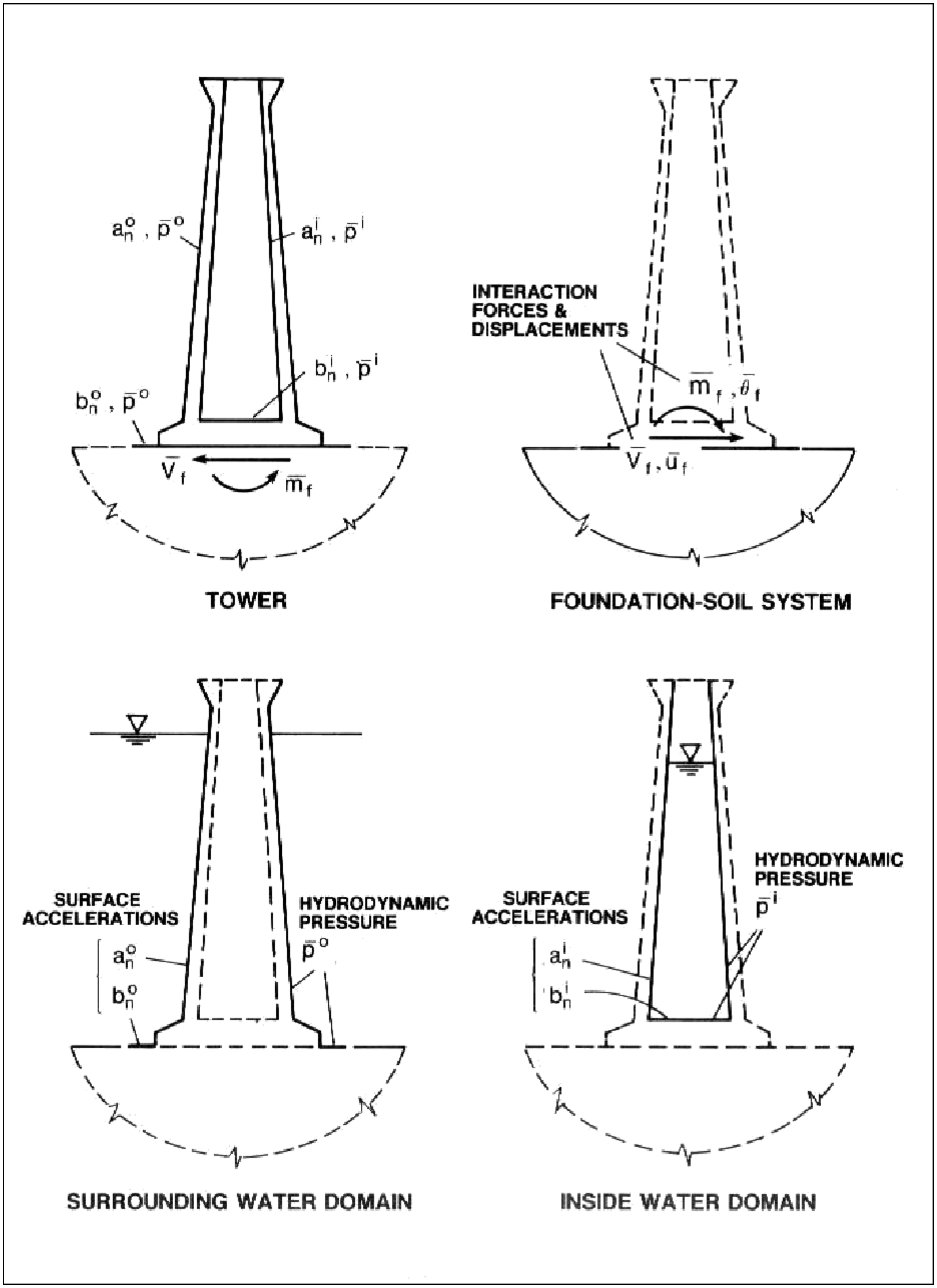


Figure 2-8. Substructure representation of tower-water-foundation soil system (from Goyal and Chopra 1989)

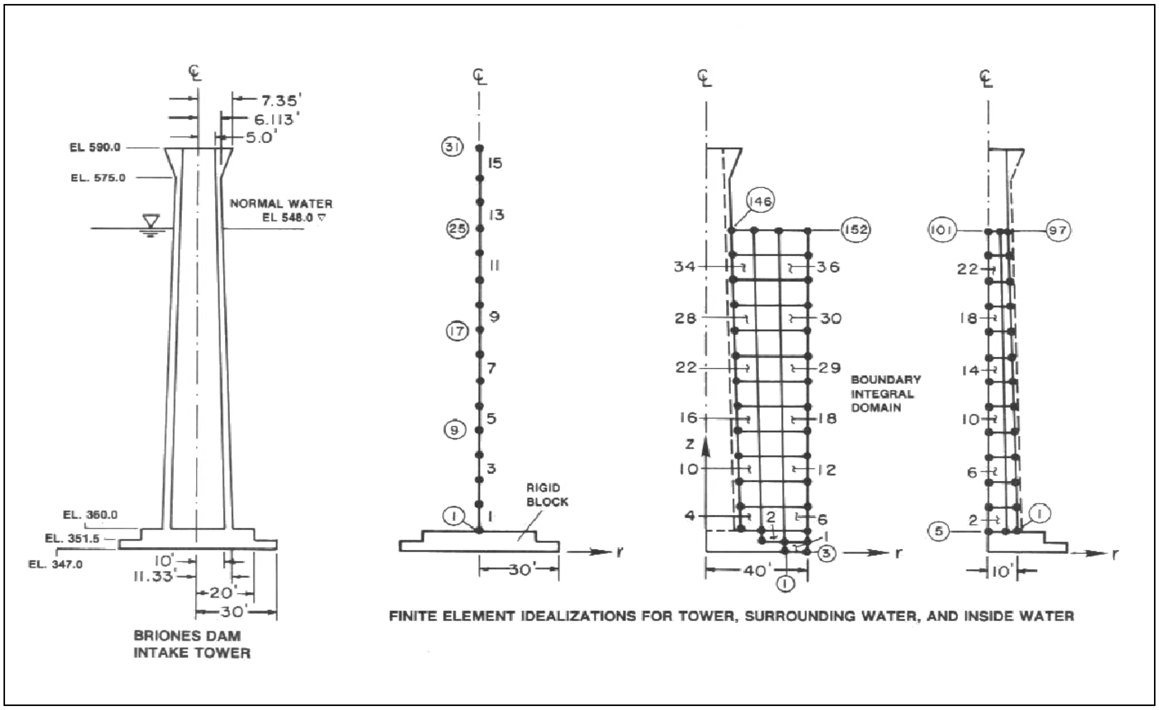


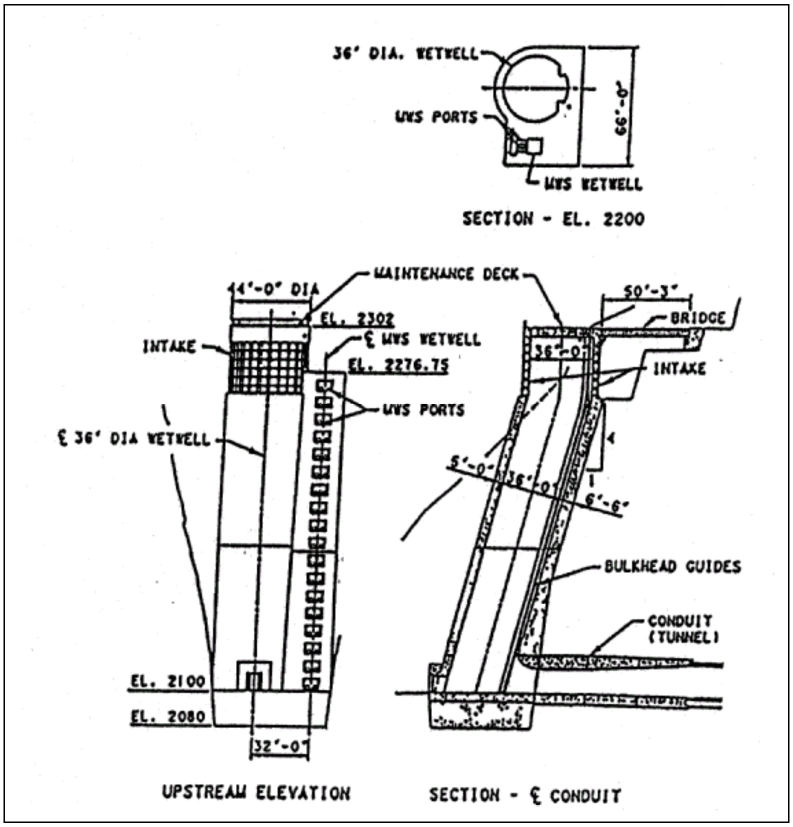
Figure 2-9. Finite element idealization of Briones Dam intake tower (from Goyal and Chopra 1989)

(b) The lateral hydrodynamic forces and external hydrodynamic moments due to pressures on the outside and inside surfaces of the tower are determined by the finite element coupled with the boundary integral procedure described in paragraph 2-20b. The fluid domain between the outside surface of the tower and a hypothetical cylindrical surface is discretized by finite elements, and the effects of the fluid domain beyond the hypothetical surface are treated by boundary integral procedures (Figure 2-9). The analysis of inside water for determining the lateral hydrodynamic forces and external moments acting on the inside face of the tower is carried out by the finite element discretization, as shown in Figure 2-9. The solution neglects the effects of surface wave and water compressibility, but has been shown to be valid for towers with a wide range of slenderness ratios and excitation frequencies (Liaw and Chopra 1973, 1974).

(c) The earthquake excitation for the tower-water-foundation system is defined by two horizontal components of the free-field ground acceleration. The vertical component of ground motion is expected to have little influence on the response of towers and is therefore not considered in the analysis. The ground motion is assumed to be identical at all points on the horizontal base of the tower. The dynamic response of the tower for each horizontal component of ground motion can be evaluated separately and the responses to the two components superimposed to determine the total response.

b. Supported towers

(1) For dams in regions of high seismicity where a free-standing tower may not be feasible, an intake structure supported against the abutment should be considered. Two examples of supported towers are the inclined intake structure for the Seven Oaks Dam by U.S. Army Engineer District, Los Angeles, and the vertical intake/outlet tower for the Eastside Reservoir Project by the Metropolitan Water District of Southern California. The Seven Oaks intake structure, shown in Figure 2-10, is a reinforced concrete intake tower inclined against the abutment and partially embedded into the rock formation. Located in southern California, the inclined tower was anchored to the rock slope to withstand the earthquake forces generated by OBE and MCE events.



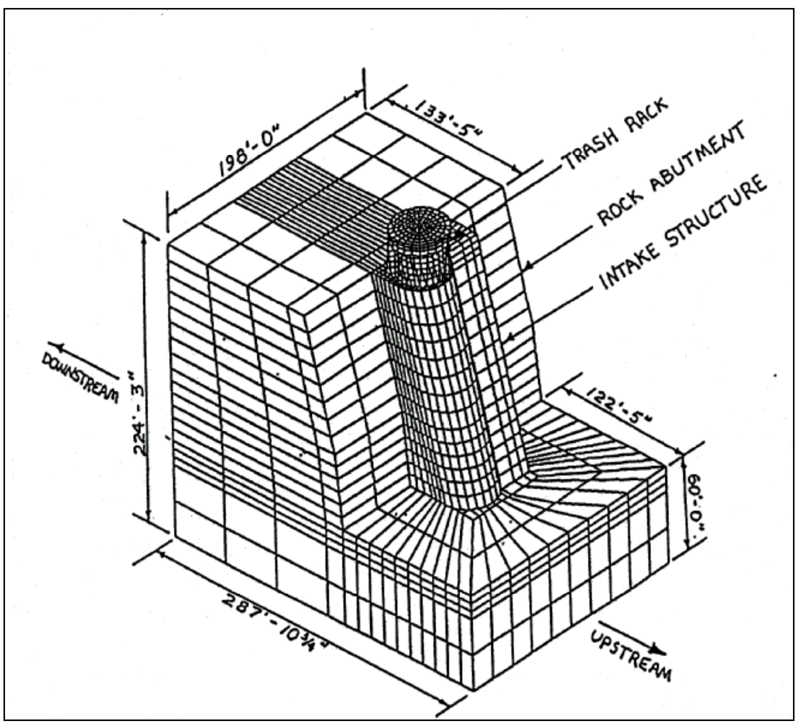


Figure 2-10. Seven Oaks Dam intake structure and associated finite element idealization (U.S. Army Engineer District, Los Angeles, 1992)

(2) The 3-D geometry of supported towers combined with severe seismic demand and substantial hydrodynamic forces influenced by the complicated site topography requires a 3-D finite element treatment of the structure. In addition, the nonuniform distribution of mass and stiffness and the irregular geometry of the tower cross section usually produce complicated bending and torsional behavior that can be captured only by a 3-D idealization of the structure. Generally, supported towers should be discretized using a combination of 3-D solid and shell elements or simply by 3-D solid elements. However, some components of the tower such as the trashrack may be modeled more appropriately using beam elements. The tower-foundation interaction may be considered by including an appropriate volume of the surrounding abutment-foundation rock region as part of the finite element model of the tower. The foundation model should be developed to have dimensions approximately equal to the height of the tower in all three directions. In practice, the massless foundation rock model discussed in paragraph 2-24*a* is usually adequate, while foundation soil models may be represented by equivalent springs or by a combination of springs and dashpots described in paragraph 2-25*a*. Figure 2-10b shows a finite element model of Seven Oaks intake structure constructed exclusively of 8-node solid elements, except that a combination of 3-D solid and beam elements was used in idealization of the trash rack. The seismic input for analysis of the supported towers includes three components of ground acceleration time-histories applied at the fixed exterior nodes of the foundation model.

(3) The effect of hydrodynamic pressures on supported towers is represented by the equivalent added-mass concept (paragraph 2-19). The added hydrodynamic mass for these towers, however, depends not only on the geometry of the tower but also on topography of the surrounding abutment-foundation region and should be computed using the finite element or boundary element procedures described in paragraph 2-20*c*. Figure 2-24 (paragraph 2-20*c*) is an example of the boundary element added-mass model applied to the analysis of Seven Oaks intake tower.

2-15. U-frame and W-frame Navigation Locks

The interactions with the soil-pile foundation, backfill soil, and the contained water can significantly affect dynamic response of lock structures. These effects should therefore be modeled with reasonable accuracy in the time-history analysis. In most cases water-structure interaction is adequately represented by the added hydrodynamic mass described in paragraph 2-15*a*(3). The SPSI effects can be incorporated in the analysis by two different approaches: direct method and substructure method. In the direct method, a complete model of the soil-pile-structure system is developed and subjected to a prescribed input motion. For lock structures whose monoliths are supported by several hundred piles, the 3-D direct method of SPSI analysis may not be feasible. In these situations, a series of 2-D approximations of the complete soil-pile-structure system is required in order to capture the 3-D effects. The 2-D approximation should be attempted for both the upstream-downstream and the cross-stream directions. In the substructure method, the lock structure and the foundation soil including the piles are treated separately. The structure is modeled using the standard finite element method. The soil-pile foundation is represented by impedance functions in the form of springs and dashpots attached to the base of the structure. The earthquake response of the structure is then computed by subjecting the soil-pile-structure system to a foundation-input motion.

a. Direct method. The 2-D direct method of SPSI analysis can be carried out using the computer program FLUSH (Lysmer et al. 1975). In this program, the response computation is carried out in the frequency domain and then the results are converted into time domain by the inverse Fourier transformation. The method of response computation in the frequency domain is described in Chapter 3.

(1) Lock model. A cross section of the lock monolith is idealized using primarily 2-D solid concrete elements. Rigid links and beam elements may be required for modeling the rigid connections and the beam-like components. Minor voids and stiffness irregularities present in the lock monolith may be represented by a *smeared* 2-D model in which the mass and stiffness of corresponding elements are adjusted (reduced) to

account for such effects. Major voids may still be smeared in 2-D elements, but upper and lower bound models consisting of solid and partially hollow sections may also be required. The model of a bridge monolith, which supports an access bridge, should include the bridge piers as part of the finite element model of the lock, as shown in Figure 2-11. The mass of the bridge deck, if significant, may simply be lumped at the top of the piers.

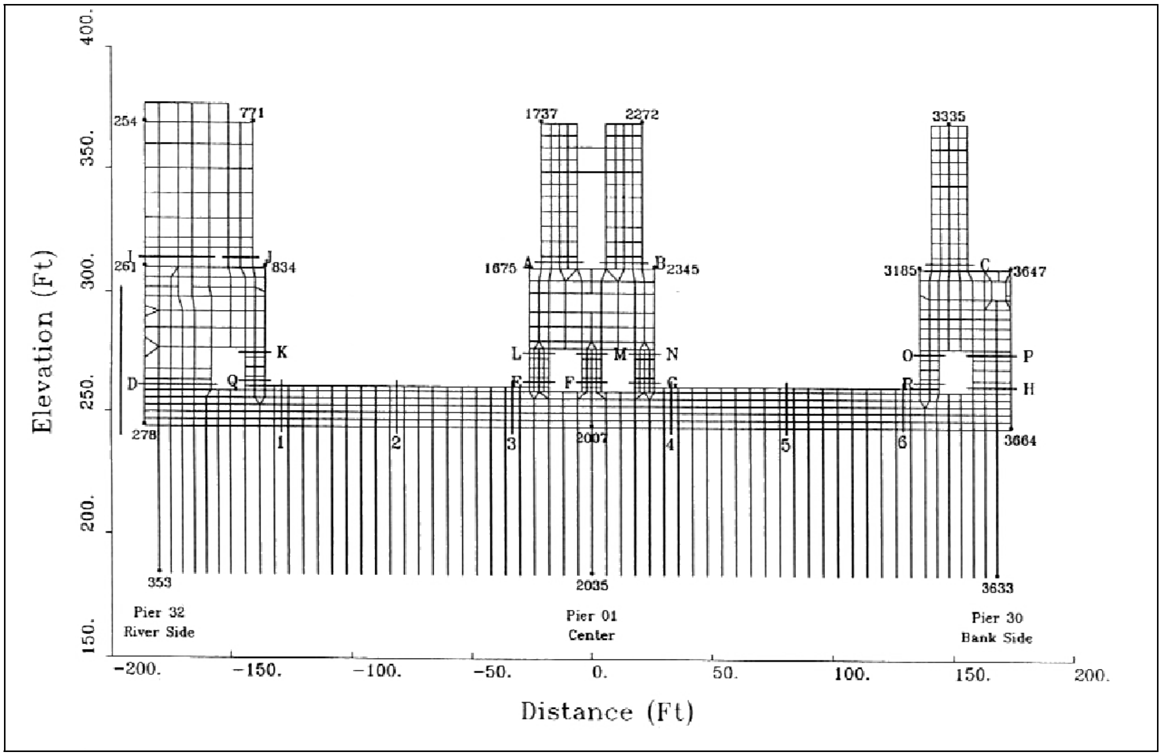


Figure 2-11. Finite element idealization of bridge pier monolith and associated H-piles for Olmsted Locks (U.S. Army Engineer District, Louisville, 1994a)

(2) Soil-pile foundation model

(a) For the SPSI analysis, each individual pile is modeled by a series of beam elements and the supporting soil is represented by plane-strain 2-D solid elements (Figure 2-12). To simulate the rigidity of the connection between the pile head and the basemat of the lock structure, the pile elements should be extended a minimum of 0.9 m (3 ft) into the basemat or special provisions adapted to ensure the moment transfer between the concrete solid elements and the pile beam elements. The vertical dimension of the soil elements should be selected carefully, because it controls the maximum frequency of the motion that can be retained in the analysis. As suggested by Lysmer et el. (1975) the vertical dimension of the soil elements should not be greater than 1/5 of the shortest wavelength of interest. The shortest wavelength of interest is defined as the ratio of the lowest shear wave velocity to the maximum frequency to be retained in the analysis. The horizontal dimension of soil elements can be selected several times the vertical dimension and may be governed by the pile spacing and other conditions.

(b) The soil model should also include a thin soft layer of about 76 mm (3 in.) beneath the lock structure in order to simulate the seismic loads being carried primarily by piles, as required by EM 1110-2-2906. Another reason for including a thin soft layer or gap beneath the lock is because complete fixity between the

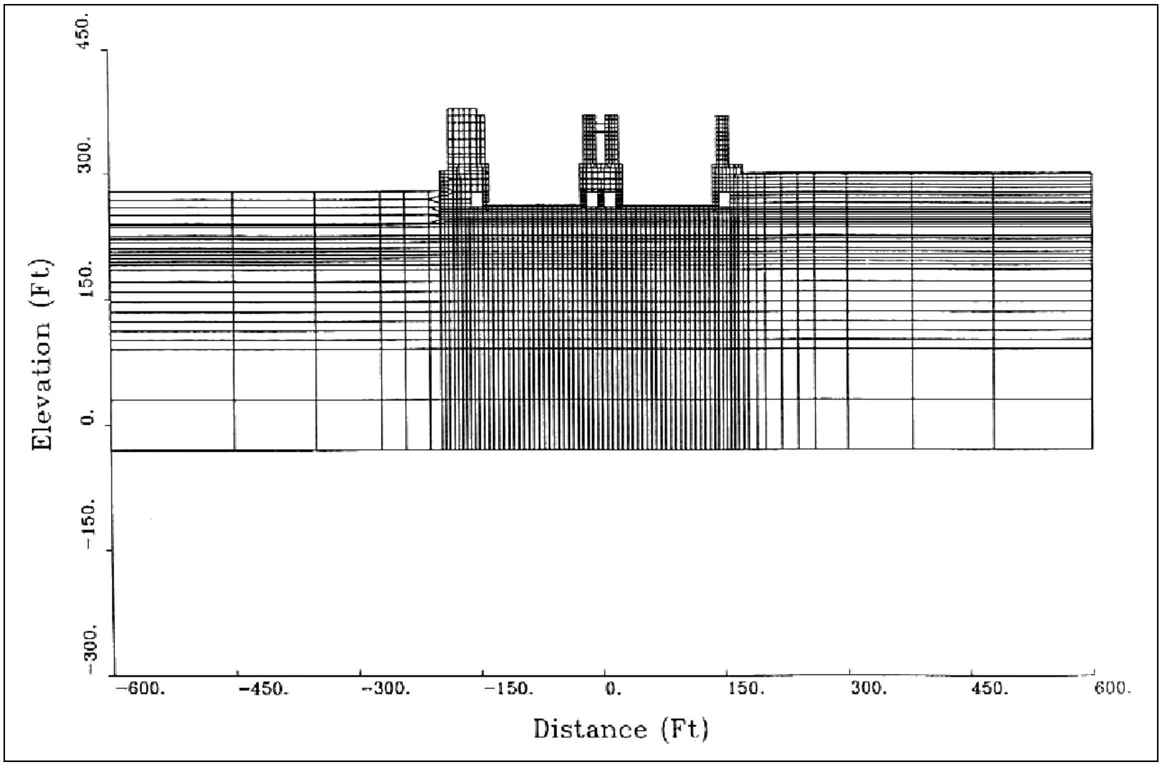


Figure 2-12. Finite element representation of bridge pier monolith, soil-pile foundation, and backfill soils for Olmsted Locks (U.S. Army Engineer District, Louisville, 1994)

lock and underlying soil either does not exist or is diminished during the ground shaking. It is therefore necessary to consider such a possibility to ensure a conservative design with sufficient numbers of piles.

(c) Another important aspect of the soil-pile foundation analysis is the appropriate characterization of the dynamic soil properties. Both the shear modulus and soil damping affect the foundation response and the soil-structure interaction. These parameters need to be defined in terms of both the low-strain shear modulus and damping and their variation with shearing strain. The large shear deformations that occur in soils during strong earthquakes introduce significant nonlinear behavior in the foundation soil region that must be accounted for in the analysis. In the frequency domain SPSI analysis, which is limited to the linear viscoelastic system, the nonlinear soil response is approximated by the equivalent linear method (Seed and Idriss 1969). According to this method a linear analysis can provide an approximate nonlinear solution if the stiffness and damping used in the analysis are compatible with the level of shear strains developed in the soil. Seed and Idriss (1970) provide data on variation of shear modulus and damping with shear strain for typical clays and sands.

(3) Water-structure interaction model.

(a) Earthquake ground motions generate two types of hydrodynamic pressures in a lock structure—impulsive and convective. The impulsive pressure represents the effects of that portion of the water that moves in unison with the lock; the convective pressure represents the effects of the sloshing action of the water. The impulsive pressures exerted on the lock walls during earthquake ground shaking are computed using the velocity potential method described in paragraph 2-19*c*. For the purpose of computing hydrodynamic pressures, and only for this purpose, the lock walls are assumed to be rigid. The pressure distribution obtained in this manner is converted into nodal lumped masses according to the tributary area associated with each node. These hydrodynamic masses are then considered as additional nodal masses in the earthquake analysis of the lock. The added hydrodynamic mass should be computed for all lock walls subjected to the hydrodynamic pressures during earthquake excitation. For example, the added hydrodynamic mass for a w-frame lock (Fig-ure 2-11) may be required for the inside surface of the land wall, both sides of the center wall, and the inside and outside surfaces of the river wall. Impulsive pressures are also exerted on the lock floor, but the added hydrodynamic masses associated with the floor pressures must be considered only in the vertical direction.

(b) Convective pressures associated with the water sloshing are evaluated as described in paragraph 2-19*c*(2). The fundamental period of water sloshing for a typical navigation lock is usually several seconds long. At such long periods the generated hydrodynamic pressures are probably as much as two orders of magnitude smaller than the impulsive hydrodynamic pressures. The effect of water sloshing on the response of navigation locks is therefore negligible and may be ignored.

b. Substructure method. In the substructure method of analysis the lock structure and the foundation soil including piles are treated separately. Similar to the direct method, the lock structure is idealized by the standard finite element method as described in *a*(1) above. The soil-pile foundation is represented either by impedance functions in the form of frequency-dependent springs and dashpots or by simple frequency-independent springs attached to the base of the structure as described in the following paragraphs.

(1) Impedance function (dynamic stiffness). The impedance functions are complex frequency-dependent coefficients whose real part can be visualized as a frequency-dependent spring coefficient and the imaginary part as a frequency-dependent damping coefficient. Also known as the dynamic stiffness, the impedance functions are computed by analytical or numerical solutions. The numerical solution of the dynamic stiffness of the soil-pile foundation may involve development of a model similar to that described in *a*(2) above. Usually the foundation mat is assumed to be rigid and the piles embedded in a layered soil medium. The properties of each soil layer and each pile are characterized by their corresponding modulus of elasticity, mass density, Poisson's ratio, and material damping. The solution is carried out in the frequency domain by applying unit harmonic loads at the structure-foundation interface nodes and obtaining the displacements or flexibility coefficients at the corresponding degrees of freedom. The dynamic stiffness is then computed by inverting the dynamic flexibility. Note that all the operations must be repeated for each frequency. Incorporation of the dynamic stiffness in the subsequent structural analysis depends on the capabilities of the computer program selected for the analysis. If the computer program selected for the subsequent structural analysis can perform response analysis in the frequency domain, the dynamic stiffness can be used directly as the input. Otherwise the impedance function needs to be approximated at or near the fundamental frequency of the system by frequency-independent stiffness, mass, and damping coefficients.

(2) Frequency-independent spring, mass, and damping. The impedance function may be approximated by frequency-independent stiffness, mass, and damping at or near the fundamental frequency of the system using regression or curve-fitting procedures. The real part of the impedance function represents mainly the stiffness and inertia of the foundation, and the imaginary component reflects the radiation and material damping. The real part Re may be approximated by:

$$Re\left[I(\omega)\right] = K - \omega^2 M \tag{2-4}$$

where

$I(\omega)$ = impedance function at frequency ω

K = stiffness coefficient

M = mass coefficient

The imaginary part of the impedance function *Im* is approximated by:

$$Im\left[I(\omega)\right] = i\omega C \tag{2-5}$$

where C is the damping coefficient. Using these approximate relationships the spring, mass, and damping coefficients corresponding to each structure-foundation interface DOFs are selected such that they would reproduce as closely as possible the actual impedance function. The procedure usually provides a good fit in the low and medium frequency ranges, but the inertia term leads to a significant error in the higher frequencies. Although this approximation provides frequency-independent lumped parameters, the structural analysis program should have capabilities for including dashpots at selected DOFs.

2-16. Massive Concrete Lock Walls

a. Massive walls with no backfill. The time-history dynamic response of a massive lock wall with no backfill soil can be evaluated using the standard finite element procedures described in paragraph 2-12*a*(2) for concrete gravity dams. A 2-D model of the wall section and the associated foundation rock is developed as described for gravity dams and illustrated in Figure 2-3b. The hydrodynamic pressures exerted on the lock wall are computed using the velocity potential method discussed in paragraph 2-19*c*. The resulting pressure distribution is then converted into equivalent nodal masses according to the tributary area associated with each node. Details of the finite element modeling of the wall section and the foundation rock and the application of seismic input should closely follow those of the gravity dams.

b. Massive walls with backfill. The time-history response evaluation of lock walls with backfill soil is more complicated because in addition to the interaction with the foundation rock and water, the wall is also subjected to the dynamic soil pressures induced by ground shaking. In this situation an accurate evaluation of the dynamic soil pressures is essential to the seismic analysis and design of the lock wall. The available methods of analysis of the backfill soil pressures fall into three categories according to the expected movement of the backfill and wall during seismic events.

(1) Yielding backfill. The relative motions of the wall and backfill material are sufficiently large to induce a limit or failure state in the soil. Representative of this approach is the well-known Mononobe-Okabe method (Mononobe and Matuo 1929; Okabe 1924) and its several variations, in which a wedge of soil bounded by the wall and an assumed failure plane are considered to move as a rigid body subjected to the same ground acceleration. The dynamic soil pressures using this approach are obtained as described in Ebeling and Morrison (1992). The resulting dynamic pressures expressed in terms of an equivalent force are then considered as an external load in the time-history analysis of the wall-foundation system as described in *a* above.

(2) Nonyielding backfill. For sufficiently low intensity ground motions the backfill material may be considered to respond within the linear elastic range of deformations. Under this condition the shear strength of the soil is not fully mobilized and the backfill is said to be nonyielding. The dynamic soil pressures and associated forces for a nonyielding backfill are computed on the basis of elastic response. The dynamic pressures of a backfill idealized as a semi-infinite uniform soil layer can be computed using a constant-parameter SDOF model (Veletsos and Younan 1994) or a more elaborate frequency-independent, lumped-parameter, MDOF system (Wolf 1995). The dynamic pressures for a more general backfill soil condition can be determined by the finite element method discussed in (3) below.

(3) Intermediate case. The intermediate case in which the backfill soil undergoes nonlinear deformations can be represented by the finite element procedures using a soil-structure-interaction computer program such as FLUSH (Lysmer et al. 1975). As illustrated in Figure 2-13, a SPSI model of the wall section should include interaction with the foundation rock, backfill soil, and the water. The wall section is idealized primarily using 2-D solid concrete elements. The foundation rock is represented by plane-strain 2-D solid elements with modulus, Poisson's ratio, and unit weight appropriate for the rock. Transmitting boundaries in the form of dashpots are introduced at the sides of the foundation rock to account for the radiation damping. The backfill soil is modeled using plane-strain 2-D soil elements. The model should account for the variation of soil properties with depth and the material nonlinear behavior of soil. The shear modulus and soil damping vary with level of shearing strain. The nonlinear behavior is usually approximated by the equivalent linear method. The unbounded boundary condition of the backfill soil may be represented by dashpots. The hydrodynamic pressures exerted on the lock wall are computed using the velocity potential method discussed in paragraph 2-19*c*.

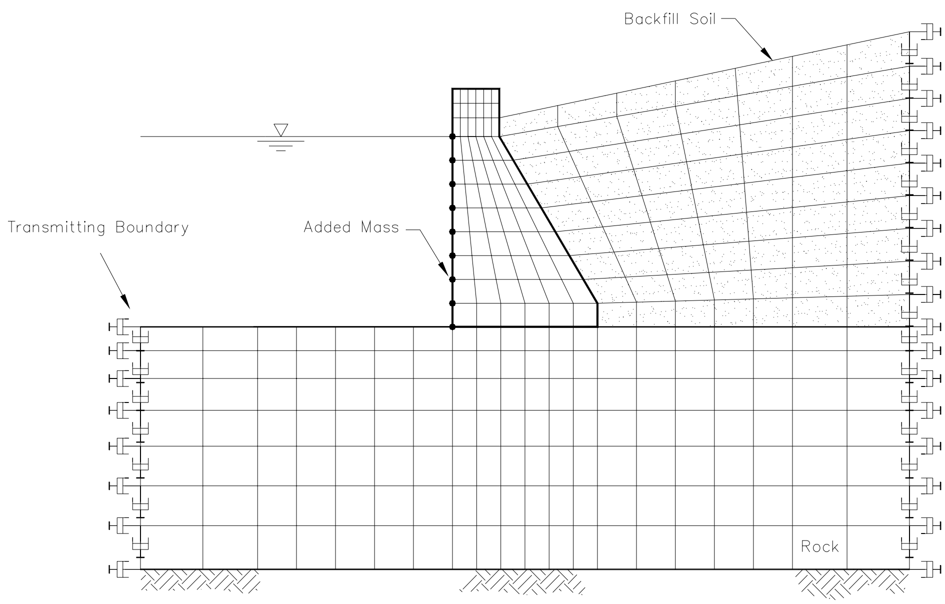


Figure 2-13. Finite element discretization of concrete lock wall, foundation rock, and backfill soil with transmitting boundaries

2-17. Massive Concrete Guide Walls

The time-history analysis of a fixed guide wall supported on cells or drilled shafts is best represented by a 3-D soil-structure-interaction (SSI) analysis. Such analysis can be performed using the computer program SASSI (Lysmer et al. 1981) or other programs and methods with similar capabilities. Since the precast concrete beams placed on the wall are not structurally connected to the wall, the 3-D model may include only one cellular pile or drilled shaft as shown in Figure 2-14. The model may be developed for analysis in the critical transverse direction (i.e., perpendicular to long axis). Whenever possible the model should take advantage of the structural symmetry to reduce the size of the problem and make the numerical solution more manageable.

The overall model usually consists of various components including the cellular sheet pile, soil layers and soil

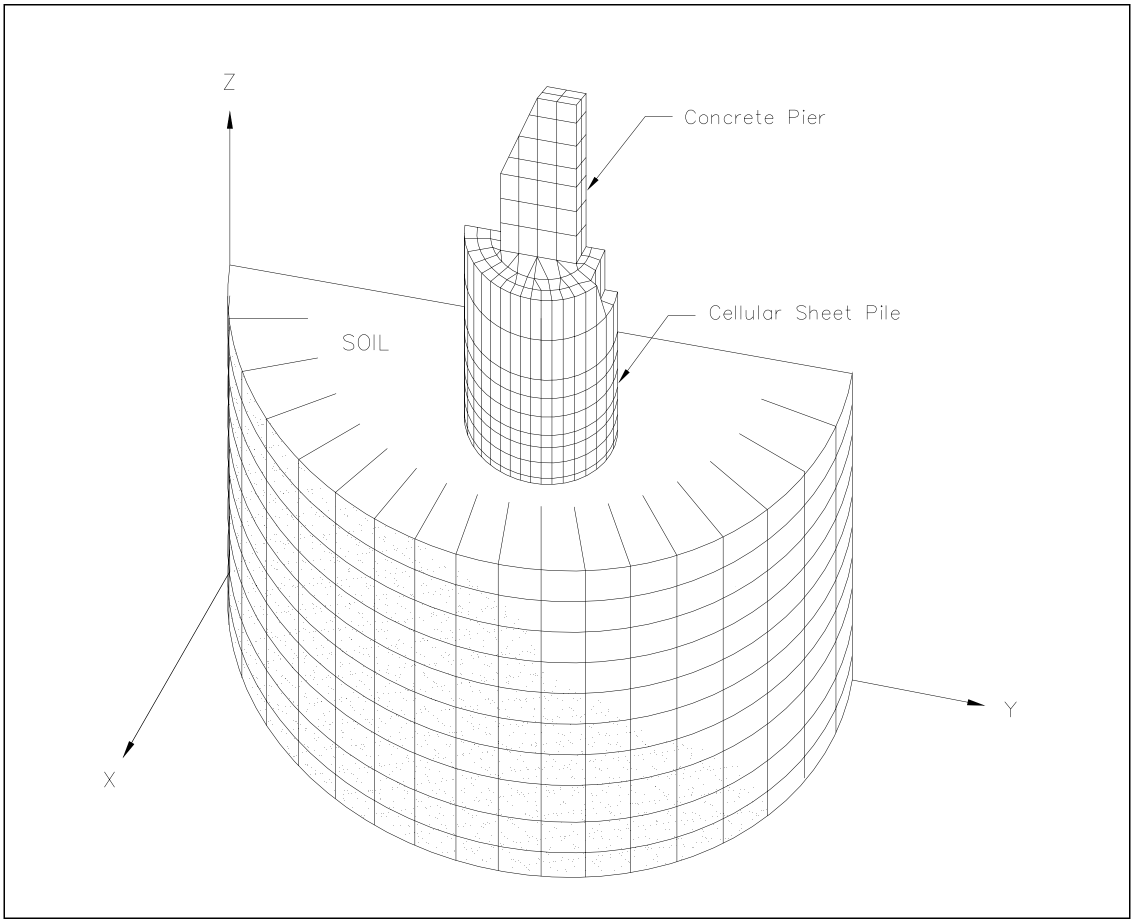


Figure 2-14. Three-dimensional finite element idealization of a fixed guide wall supported on cellular sheet pile

within the sheet pile, concrete pier or block above the ground surface, precast concrete beams, and the hydrodynamic forces acting on the pile and the concrete block.

a. Sheet piles. The cellular sheet piles are modeled using plate or shell elements. Each sheet pile is represented by a series of plate/shell elements connected horizontally and vertically to each other and at top to the concrete block. The elastic modulus of the plate/shell elements can be obtained from the work of Mosher (1992). Mosher has concluded that the field experimental data indicate that due to imperfect contact along the interlock between two adjacent sheet piles, the interlock initially undergoes significant movement but gradually stiffens after some movements, thus resulting in reduced subsequent lateral movements. The correlation studies by Mosher have also shown that the sheet piles modeled by plate/shell elements having a horizontal-to-vertical modulus ratio of 0.03 result in a good agreement between the computed and measured interlock forces. This research shows that orthotropic material properties are required for the plate/shell elements. Since SASSI and probably other SSI programs do not have orthotropic plate element capabilities, vertical beam elements can be used to compensate and represent the vertical stiffness of the sheet piles. That is, the low stiffness based on the modulus ratio of 0.03 is used for the plate/shell elements to simulate the interlock action along the horizontal planes, and vertical beam elements are employed to represent the vertical stiffness of the sheet piles. The mass and bending stiffness of the beam elements are set to zero, because only the axial stiffness of beam elements is needed.

b. Soil model. Both the soil surrounding the structural model shown in Figure 2-14 and that confined within the cellular sheet piles should be incorporated in the computer model. As shown in this figure, the structural model is embedded in the foundation soil. The free-field soil at the site may consist of several soil layers having different dynamic properties including the unit weight, shear modulus/shear wave velocity, Poisson's ratio/compression wave velocity, and the damping. The dynamic properties of the soil confined within the cell should be selected by giving due consideration to the effects of shaking on the strain-dependent nature of soil properties and the mechanism of load transfer from the sheet piles to the confined soil, whose properties depend on the confining pressure. For the soil confined inside the cell, it is reasonable to assume that the earthquake shaking will induce insignificant shear straining of the soil. Thus, the low-strain dynamic soil properties may be more representative for the inside soil. With regard to the effect of transfer of the sheet-pile load to the underlying soil, two cases may be analyzed. In one case the weight of sheet-pile cells is assumed to be carried directly by the soil within the cell, resulting in higher confining pressures than that due to the weight of overlying soils alone. In the second case the weight of sheet-pile cells is assumed to be carried by the sheet piles-soil interface, resulting in negligible increase in confining pressures of the inside soil. These two cases produce two different sets of dynamic soil properties that can be used in a parameter study to examine the effects of soil properties on the dynamic response of the guide wall.

c. Concrete block or pier. The structural concrete block or pier at the top of the sheet piles can be modeled using 8-node solid concrete elements. Since the precast beams are not structurally connected to the wall, only their masses may be considered and added to the mass of the concrete block or pier.

d. Hydrodynamic effects. The effects of hydrodynamic forces acting on both the cell and the precast concrete blocks are approximated by the added hydrodynamic mass concept. The hydrodynamic pressures on the circular cell can be computed based on the procedure described in paragraph 2-19*d*. The hydrodynamic pressures on the precast concrete blocks and beams are calculated by the generalized Westergaard method discussed in paragraph 2-19*b*.

Section III
Fluid-Structure Interaction

2-18. General

A hydraulic structure and water interact through hydrodynamic pressures at the structure-water interface. In the case of concrete dams, the hydrodynamic pressures are affected by the energy loss at the reservoir boundary. Generated by the motions of the structure and the ground, hydrodynamic pressures affect deformations of the structure, which in turn influence the pressures. The complete formulation of the fluid-structure interaction produces frequency-dependent hydrodynamic pressures that can be interpreted as an added force, an added mass, and an added damping (Chopra 1987). The added hydrodynamic mass influences the structure response by lengthening the period of vibration, which in turn changes the response spectrum ordinate and thus the earthquake forces. The added hydrodynamic damping arises from the radiation of pressure waves and, for dams, also from the refraction or absorption of pressure waves at the reservoir bottom. The added damping reduces the amplitude of the structure response especially at the higher modes.

2-19. Simplified Added Hydrodynamic Mass Model

If the water is assumed to be incompressible, the fluid-structure interaction for a hydraulic structure can be represented by an equivalent added mass of water. This assumption is generally valid in cases where the fluid responses are at frequencies much greater than the fundamental frequency of the structure. Following sections describe the simplified added-mass procedures including original and generalized Westergaard methods, velocity potential method for Housner's water sloshing model, and Chopra's procedure for intake-outlet towers and submerged piers and shafts.

a. Westergaard added mass. According to Westergaard (1933) the hydrodynamic forces exerted on a gravity dam due to earthquake ground motion are equivalent to inertia forces of a volume of water attached to the dam and moving back and forth with the dam while the rest of the reservoir water remains inactive. For analysis of gravity dams idealized as a 2-D rigid monolith with vertical upstream face, Westergaard proposed a parabolic shape for this body of water as shown in Figure 2-15. The added mass of water at location m_{ai} is therefore obtained by multiplying the mass density of water ρ_w by the volume of water tributary to point i:

$$m_{ai} = \frac{7}{8}\rho_w\sqrt{H(H - z_i)}A_i \tag{2-6}$$

where

H = depth of water

z_i = height above the base of the dam

A_i = tributary surface area at point i

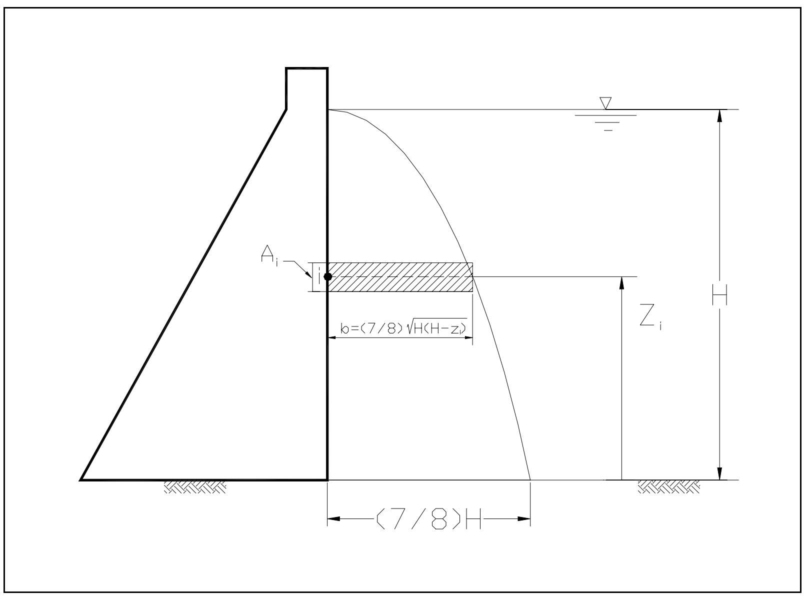


Figure 2-15. Westergaard added-mass representation

b. Generalized Westergaard added mass. Westergaard's original added-mass concept described in *a* above is directly applicable to the earthquake analysis of gravity dams and other hydraulic structures having a planar vertical contact surface with the water. For structures having sloped or curved contact surfaces, a generalized formulation of the added mass should be employed. The generalized formulation assumes that the pressure is still expressed by Westergaard's original parabolic shape, but the fact that the orientation of the pressure is normal to the face of the structure and its magnitude is proportional to the total normal acceleration at that point is recognized. In general, the orientation of pressures on a 3-D surface varies from point to point; and if expressed in Cartesian coordinate components, it would produce added-mass terms associated with all three orthogonal axes. Following this description the generalized Westergaard added mass at any point *i* on the face of a 3-D structure is expressed (Kuo 1982) by

$$m_{ai} = \alpha_i A_i \lambda_i^T \lambda_i = \alpha_i A_i \begin{bmatrix} \lambda_x^2 & & sym. \\ \lambda_Y \lambda_X & \lambda_y^2 & \\ \lambda_Z \lambda_x & \lambda_z \lambda_y & \lambda_z^2 \end{bmatrix}_i \tag{2-7}$$

where

A_i = tributary area associated with node *i*

$\lambda_i = \langle \lambda_x, \lambda_y, \lambda_z \rangle_i$ is the normal direction cosines (Figure 2-16)

α_i = Westergaard pressure coefficient given by

$$\alpha_i = \frac{7}{8} \rho_w \sqrt{H_i (H_i - z_i)} \tag{2-8}$$

For a 3-D surface such as an arch dam curved both in plan and elevation, the added-mass terms associated with a particular node form a 3×3 full matrix. However, matrices for various nodes are not coupled.

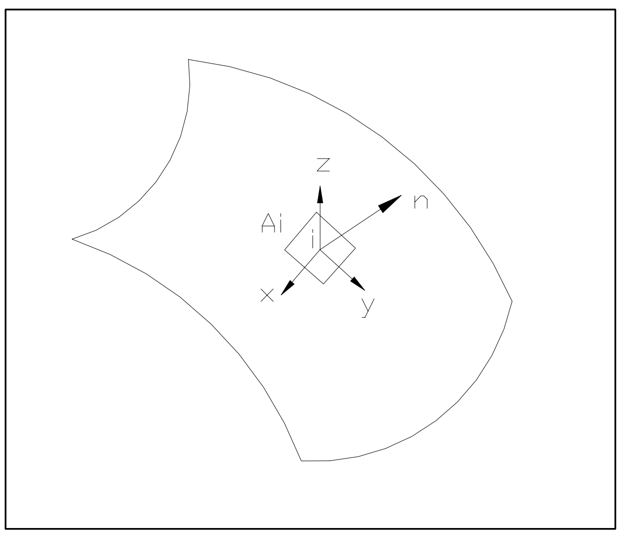


Figure 2-16. Normal and Cartesian directions of curvilinear surface

c. Velocity potential solution of added-mass navigation locks. The hydrodynamic pressures of contained water exerted on navigation lock walls due to earthquake excitation can be divided into impulsive and convective components (Housner 1957). The impulsive pressure represents the effects of the portion of the fluid that moves in unison with the lock (added mass); and the convective pressure represents the effects of the sloshing action of the fluid.

(1) The impulsive pressures due to a horizontal component of ground motion $\ddot{u}_x$ can be obtained from a velocity potential function that satisfies the Laplace equation with appropriate boundary conditions (Haroun

1984). The hydrodynamic pressure solution for a navigation lock chamber with a cross-section width of *2L* filled with water to a depth of *H* (Figure 2-17) is given by

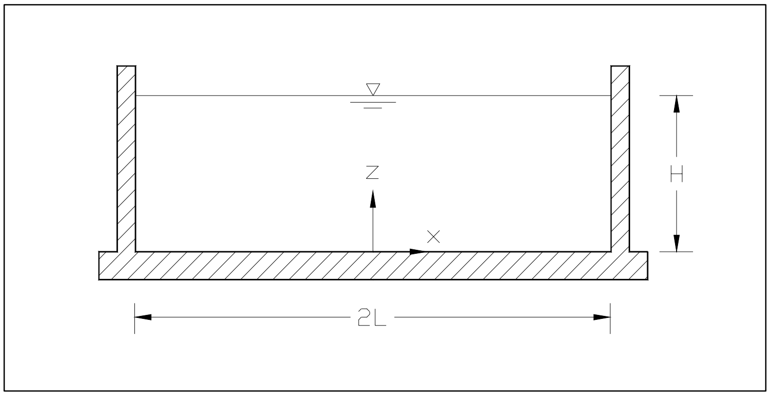


Figure 2-17. Lock chamber dimensions and coordinate system

$$P(L,y,z,t)=\frac{2\rho_w}{H}\sum_{i=1}^{\infty}\frac{(-1)^i}{\lambda_i^2}\tanh(\lambda_i L)\,\cos(\lambda_i z)\;\ddot{u}_x(t) \tag{2-9}$$

where $\lambda_i = (2i-1)\pi / 2L$ and ρ_w is the mass density of water. It should be noted that the series in Equation 2-9 converges rapidly, and only one to three terms may be needed in practical applications. Figure 2-18 compares the pressure distributions given by this equation and the pressure distribution obtained from the Westergaard method. This comparison shows that the pressure distributions given by Westergaard and the velocity potential solution are quite different, and that the difference between the two distributions increases with the *H/2L* ratio. Therefore, for analysis of lock structures the impulsive pressure distribution given by Equation 2-9 should be used for determination of the added hydrodynamic mass.

(2) Water sloshing in navigation locks. The liquid sloshing force exerted on navigation lock walls during the earthquake shaking can be obtained from Housner's mathematical model for water tanks (Housner 1957). According to Housner the sloshing force exerted on a tank wall would be the same as that exerted by a mass M_1 attached to the tank by a restraining spring K, as shown in Figure 2-19. The mass M_1 and spring constant K correspond to the fundamental mode of the oscillating fluid and are mounted at a height of h_1 to give the same moment as the fluid. The mass M_0 attached rigidly to the tank at height h_0 gives the force resultant associated with the impulsive pressure discussed previously. Formulas for these parameters initially derived by Housner (1957) and later presented by Epstein (1976) are as follows.

$$M_0=\left[\frac{\alpha}{\sqrt{3}}\tanh\left(\frac{\sqrt{3}}{\alpha}\right)\right]M \tag{2-10}$$

$$M_1=\left[\frac{0.527}{\alpha}\tanh(1.58\alpha)\right]M \tag{2-11}$$

$$h_0=\frac{3}{8}h \tag{2-12}$$

$$h_1 = \left[1 - \frac{\cosh(1.58\alpha) - 1}{(1.58\alpha)\sinh(1.58\alpha)} \right] \tag{2-13}$$

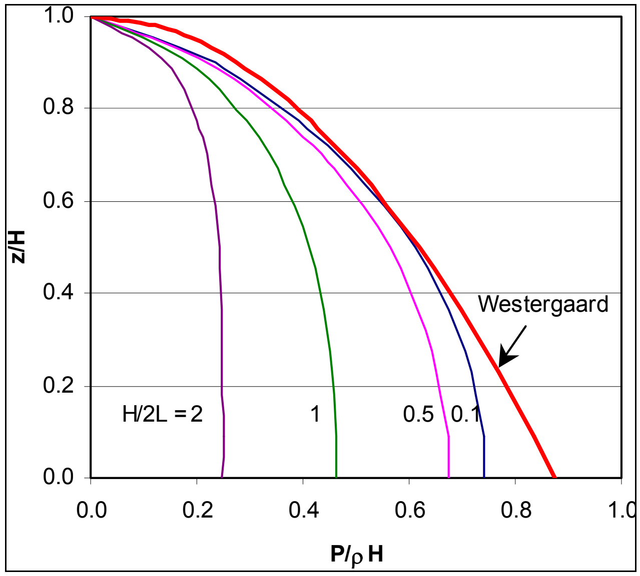


Figure 2-18. Comparison of hydrodynamic pressures for various section dimensions

where $\alpha = H/L$ and M is the total mass of water for a unit width section of the tank or lock. The period of fundamental mode of sloshing T is given by

$$T = 2\pi \left[\frac{L}{1.58g \tanh(1.58\alpha)} \right]^{1/2} \tag{2-14}$$

where g is the acceleration of gravity. Knowing the mass M_1 and period T, the sloshing force P_1 as a function of time t is obtained from

$$P_1 = M_1 S_a \sin\left(\frac{2\pi}{T}\right)t \tag{2-15}$$

where M_1S_a represents the maximum sloshing force and S_a is the spectral acceleration at period T obtained from the response spectra of earthquake ground motion. These equations are applicable for shallow tanks, i.e., $\alpha = H/L$ # 1.5, which should be valid for most navigation locks. For the cases $\alpha > 1.5$, refer to Epstein (1976).

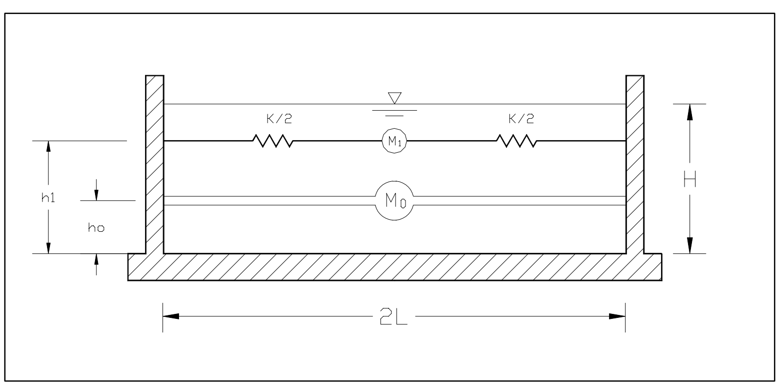


Figure 2-19. Housner's mathematical model for impulsive and convective (sloshing) hydrodynamic forces

d. Added mass for towers and supporting shafts.

(1) The hydrodynamic interaction effects of the surrounding water in the analysis of intake/outlet towers and submerged shafts and piers are approximated by an equivalent added mass of water. The procedure is based on the assumption of a rigid structure surrounded by incompressible water. The inertial effects of water, therefore, are represented by added-mass functions computed for unit horizontal ground acceleration. These added-mass functions are available only for circular cylindrical towers (Jacobson 1949; Liaw and Chopra 1973; and Rashed 1982) and for uniform elliptical towers (Kotsubo 1965). For towers and piers of arbitrary cross section having two axes of symmetry, the added mass is obtained from analysis of an "equivalent" circular tower in accordance with a simplified procedure developed by Goyal and Chopra (1989).

(2) The added mass for circular cylindrical towers or piers surrounded by water is obtained from an analytical solution of the Laplace equation, as presented in Figure 2-20 and various charts and tables by Goyal and Chopra (1989). The normalized added mass for a uniform tower of arbitrary cross section is obtained by converting the arbitrary section first into an equivalent elliptical and then into an equivalent circular section for which Goyal and Chopra's chart and tables can be used (Goyal and Chopra 1989). The procedure is also extended to the added mass analysis of nonuniform towers, simply by applying these steps to various portions of the tower or pier that actually are, or assumed to be, uniform.

(3) The added hydrodynamic mass associated with the water inside a tower is also computed from the solution for an equivalent circular section in a manner similar to that described for the surrounding water. For the equivalent circular cylindrical towers the added mass of contained water is also obtained from an analytical solution of the Laplace equation, as presented in Figure 2-21 and various charts and tables by Goyal and Chopra (1989).

2-20. Finite Element Added Hydrodynamic Mass Model

The simplified added hydrodynamic mass concept described in paragraph 2-19 is generally not appropriate for refined analysis of hydraulic structures having complex geometry such as arch dams and irregular intake/outlet towers. For such structures a finite element idealization of the fluid domain permits a more realistic treatment of the complicated geometry of the structure-water interface as well as the reservoir bottom. Assuming water is incompressible, inviscid, and irrotational, the small-amplitude motion of water is governed by the wave equation

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=0 \qquad (2\text{-}16)$$

where *p(x,y,z)* is hydrodynamic pressure in excess of the static pressure generated by acceleration of the structure-water contact surface and acceleration of the reservoir bottom. The hydrodynamic pressures acting on the structure-water interface are obtained by solving Equation 2-16 using appropriate boundary conditions. Neglecting the effects of surface waves, which are known to be small, the boundary condition at the free surface is:

$$p(x,y,z)=0 \qquad (2\text{-}17)$$

On the structure-water contact surface, where the normal acceleration $\ddot{u}_{nd}$ (Figure 2-22) is prescribed, the boundary condition becomes:

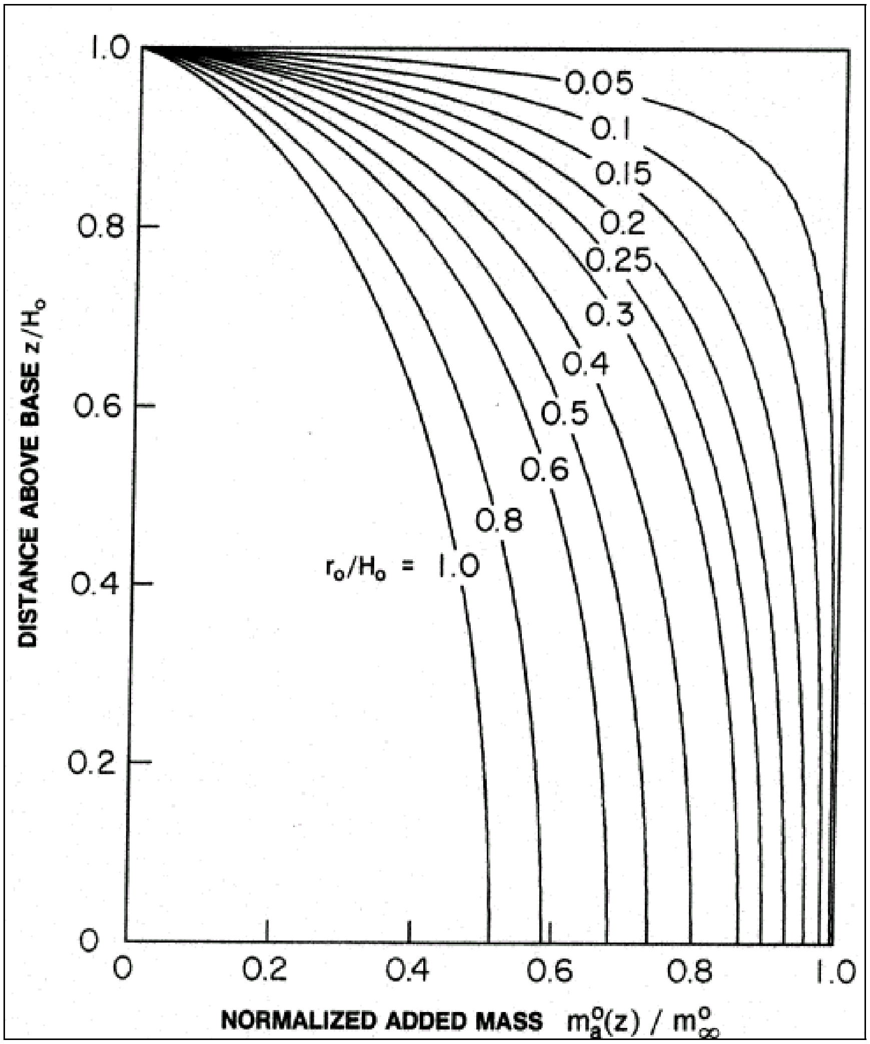


Figure 2-20. Normalized added hydrodynamic mass for circular cylindrical towers associated with surrounding water where *z* = distance above the base of the tower or pier, *H₀* = depth of the surrounding water, *r₀* = radius of the outside surface of the tower, $m_\infty^o = \rho_w \pi r_o^2$ = the added mass per unit height of an infinitely long uniform tower of the same radius, and $m_a^0(z)$ = the added mass for circular tower or pier surrounded by water (from Goyal and Chopra 1989)

$$\frac{\partial p}{\partial n} = -\rho_w \ddot{u}_{nd} \tag{2-18}$$

where n represents the direction normal to the surface. A similar boundary condition may also be applied at the reservoir boundary

$$\frac{\partial p}{\partial n} = -\rho_w \ddot{u}_{nb} \tag{2-19}$$

where $\ddot{u}_{nb}$ is the normal component of ground acceleration at the reservoir boundary (Figure 2-22). In addition to these boundary conditions, *p(x,y,z)* should remain bounded in the far field of the unbounded fluid domain.

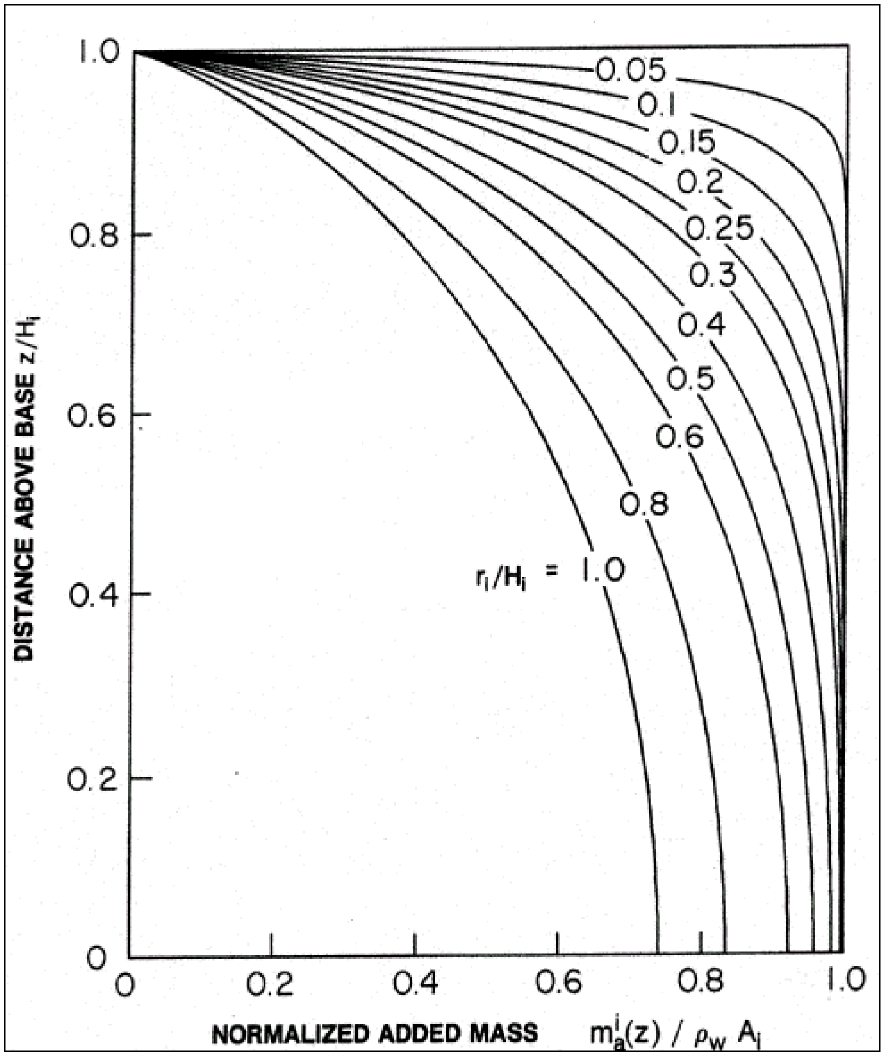


Figure 2-21. Normalized added hydrodynamic mass for circular cylindrical towers associated with inside water where *z* = distance above the base of the tower or pier, *H_i* = depth of the inside water, *r_i* = radius of the inside surface of the hollow tower, $m_\infty^i = \rho_w A_i = \rho_w \pi r_i^2$ = the mass of water contained within an infinitely long uniform tower with the radius of *r_0*, and $m_a^i(z)$ = the added mass of water contained within the circular tower (from Goyal and Chopra 1989)

a. Arch dams. For arch dams the solution of Equation 2-16 for hydrodynamic pressures is obtained numerically using the finite element method (Kuo 1982; Ghanaat 1993b), but the reservoir bottom and a truncating vertical plane at the upstream end are assumed to be rigid. This means that the ground motion $\ddot{u}_g$ is not applied to the reservoir bottom (i.e., $\ddot{u}_{nb} = 0$ in Equation 2-19) and that the radiation damping due to propagation of pressure waves in the upstream direction is not considered. The analysis involves development of a finite element discretization of the fluid domain with the truncating upstream plane located a distance at least three times the water depth from the face of the dam. At such distance, parameter studies show that the acceleration at the truncated plane has a small effect on the hydrodynamic pressures at the face of the dam and,

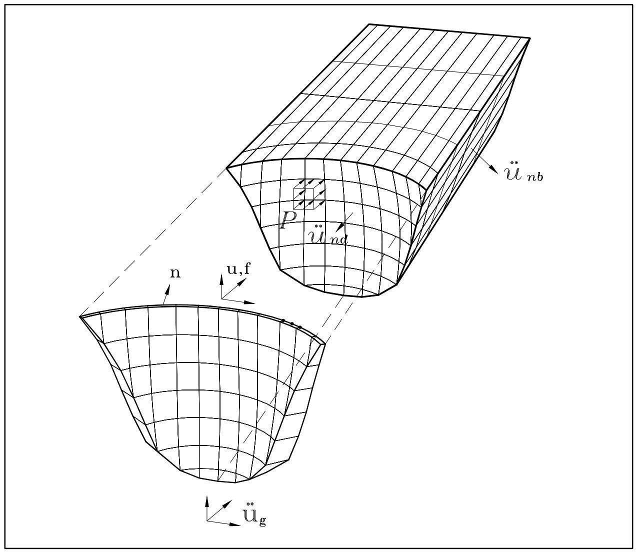


Figure 2-22. Finite element idealization of incompressible impounded water for arch dams (from Ghanaat 1993b)

thus, can be assumed zero in practical applications (Clough et al. 1984a, 1984b). In most cases a prismatic fluid mesh generated by translating the dam-water interface nodes in the upstream direction is adequate for practical purposes (Figure 2-22). However, if the actual reservoir topography is substantially different from a prismatic model, a fluid mesh that closely matches the reservoir topography may be required. In either case, the distance between the successive surfaces or planes arranged approximately parallel to the dam axis should be such that the fluid layers closer to the dam face contain finer elements. The finite element solution of Equation 2-16 results in nodal pressures on the upstream face of the dam, which after conversion into nodal forces gives the added hydrodynamic mass matrix for earthquake analysis of the dam. The resulting added-mass matrix is a full square matrix with a dimension equal to the number of degrees of freedom on the dam-water interface nodes.

b. Intake-outlet towers with two axes of symmetry. If the geometry of the tower is more complex or if the effects of the vertical acceleration of the reservoir bottom are to be considered, the simplified added mass computed on the basis of an "equivalent" circular tower may not properly represent the hydrodynamic forces acting on the tower. In such cases, the added mass for the surrounding water can be obtained from the finite element solution of Equation 2-16 together with appropriate boundary conditions at the tower-water interface, the reservoir bottom, and the free surface of water. For free-standing towers of arbitrary cross section but having two axes of symmetry, a semi-analytical procedure formulated by Goyal and Chopra (1989) may be employed. In this formulation, as illustrated in Figure 2-23, the surrounding water adjacent to the tower τ_A^o is represented by the finite element approximation, and the fluid domain beyond this immediate region τ_B^o is treated analytically using boundary integral procedures. Since the analytical solution of the boundary integral domain is evaluated for circular-cylindrical towers, the hypothetical surface Γ_c^o between the finite element and the infinite region is restricted to a circular-cylindrical surface. Note that the vertical ground motion is not included in this formulation; only the vertical acceleration on the surface Γ_e^o caused by the rotation of the foundation is considered. Similar to the surrounding water, the hydrodynamic pressures associated with the inside water are also obtained from the solution of Equation 2-16, except that boundary conditions now correspond to the tower-water interface and free surface of the inside water. This formulation has been implemented in the computer program TOWER3D (Goyal and Chopra 1989), but is restricted to 3-D response analysis of towers having two axes of symmetry in plan.

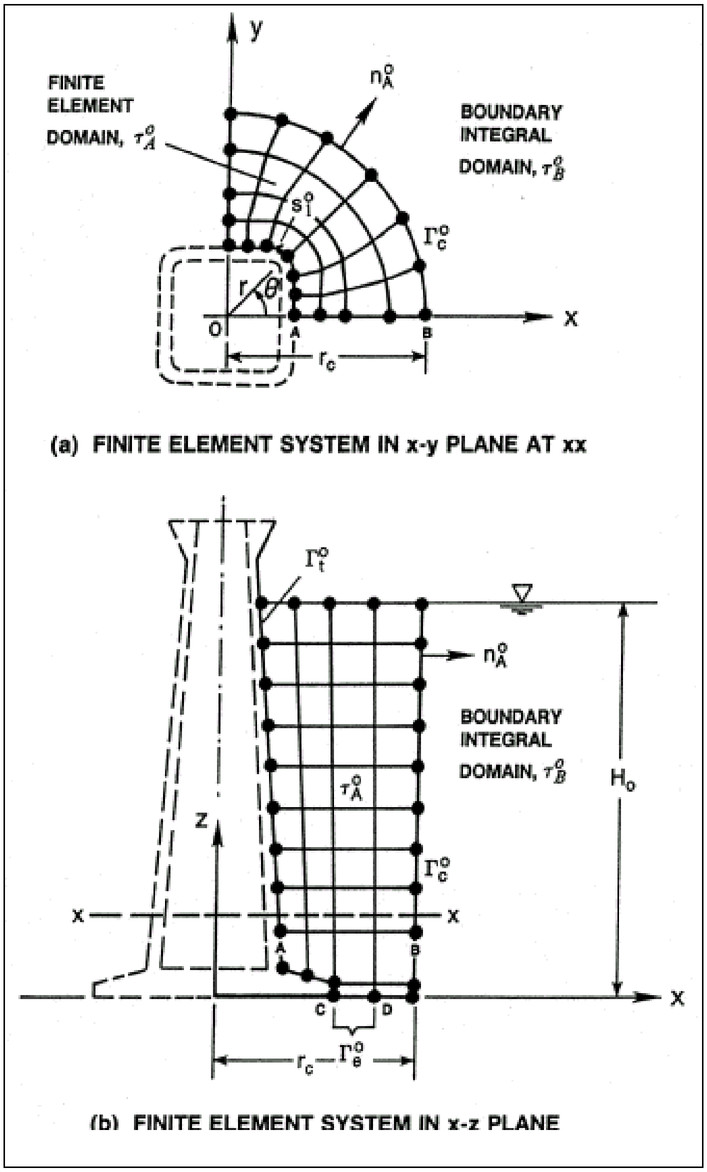


Figure 2-23. Three-dimensional finite element system for surrounding water domain (from Goyal and Chopra 1989)

c. Irregular intake towers. The added hydrodynamic mass for the analysis of irregular towers with no axis of symmetry can be obtained by the procedure described for arch dams using either the finite element or boundary element formulation. Figure 2-24 shows an example of the boundary element formulation applied to added-mass analysis of the Seven Oaks intake tower. In this approach the water partially surrounding the intake tower is represented by surface areas of the intake tower and the reservoir bottom and sides. The added-mass solution is obtained as if the entire space represented by these surface areas were filled with water. The water domain beyond the generated mesh can be assumed to extend to infinity, and the earthquake ground motion may be applied to the tower as well as to the reservoir bottom.

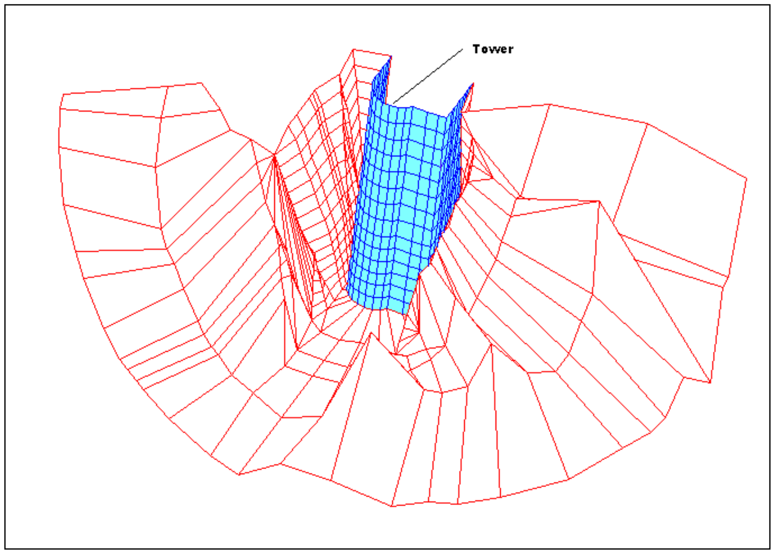


Figure 2-24. Boundary element added-mass model of Seven Oaks intake tower

2-21. Compressible Water with Absorptive Boundary Model

a. The added-mass representation of hydrodynamic pressure previously described ignores the effects of water compressibility and water-foundation interaction. Refined dam-water interaction analysis including these factors (Hall and Chopra 1980; Fenves and Chopra 1984b; Fok and Chopra 1985) indicates that water compressibility and the water-foundation interaction can significantly affect the hydrodynamic pressures and hence the response of concrete dams to earthquakes. The effects of water compressibility are generally significant when the fundamental frequency of the dam without the water is relatively close to the fundamental resonant frequency of the impounded water, $f_1^r = C/(4H)$, where C is the velocity of sound in water and H is the water depth. The water compressibility and the water-foundation interaction effects can be considered by solving the wave equation for compressible water

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{C^2}\frac{\partial^2 p}{\partial t^2} \qquad (2\text{-}20)$$

subjected to the boundary conditions given in Equations 2-17 to 2-19. The water-foundation interaction, as indicated by Equation 2-19, can be considered by using finite elements to represent the flexible foundation or modeling the foundation material as a viscoelastic half space. This effect has also been accounted for in an approximate manner by using a simplified boundary condition that models the energy dissipated at the water-foundation interface, as described in paragraph 2-22. The most extensive study of concrete dams with compressible reservoir water has been carried out by Chopra and his co-workers (Hall and Chopra 1980; Fenves and Chopra 1984b; Fok and Chopra 1985) using the substructure method of analysis. Assuming the reservoir water can be idealized as a fluid domain with constant depth and infinite length in the upstream direction, the hydrodynamic pressures for 2-D analysis of gravity dams is obtained from a continuum solution (Fenves and Chopra 1984b). For irregular reservoir boundaries, the fluid domain is usually assumed to consist of an irregular portion adjacent to the dam and a uniform section of infinite length in the upstream direction (Figure 2-25). The irregular portion is represented by a finite element discretization (Hall and Chopra 1980) or boundary element method (Humar and Jablonski 1988), whereas the uniform portion is analyzed by a continuum solution. The equal pressure conditions at the interface then enforce the coupling between the two

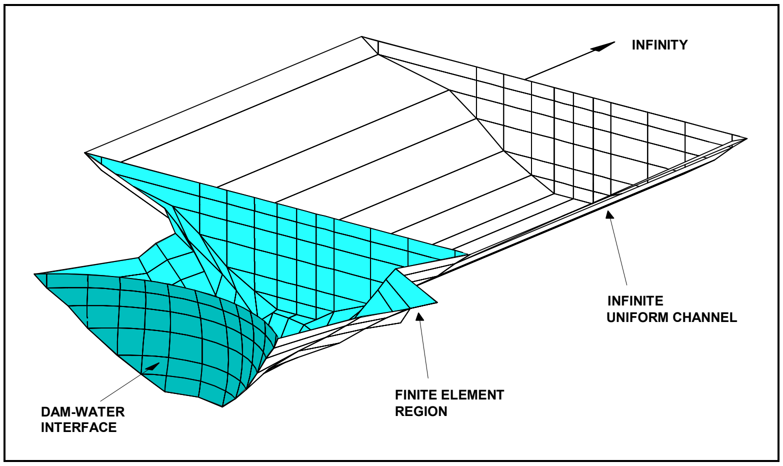


Figure 2-25. Finite element model of compressible water with absorptive boundary for Dongjiang Arch Dam, China (from Ghanaat et al. 1993)

regions. Such formulation of the hydrodynamic pressure results in frequency-dependent hydrodynamic terms that are best treated in the frequency domain. This procedure has been implemented in the computer program EACD-3D (Fok, Hall, and Chopra 1986) for the earthquake analysis of arch dams.

b. The hydrodynamic pressure in the reservoir, as given by Equation 2-20, is generated by the acceleration of the upstream face of the dam and vertical accelerations of the reservoir bottom. The solution in frequency domain produces the frequency response functions for the hydrodynamic pressures in the impounded water. The computed pressure frequency response functions at the face of the dam and at the reservoir bottom are then converted into statically equivalent nodal forces $\overline{\mathrm{R}}_h^l(\omega)$ and $\overline{\mathrm{Q}}_h(\omega)$ and are substituted into the system equations of motion (Equation 2-1).

2-22. Reservoir Boundary Absorption

a. The energy loss capability of the reservoir bottom materials is approximately modeled by a boundary that partially absorbs (refracts) the incident pressure waves (Hall and Chopra 1980). In the boundary condition for the reservoir bottom, this energy loss is represented by the damping coefficient q, which is related to the wave reflection coefficient α by

$$\alpha = \frac{1 - qC}{1 + qC} = \frac{1 - \dfrac{\rho C}{\rho_s C_s}}{1 + \dfrac{\rho C}{\rho_s C_s}} \tag{2-21}$$

where ρ and C are the density and sound velocity for water, respectively, and ρ_S and C_S are the density and sound velocity for the bottom materials, respectively. The reflection coefficient α provides a measure of the level of absorption of the reservoir bottom materials. It is defined as the ratio of the amplitude of the reflected pressure wave to the amplitude of incident pressure wave impinging on the reservoir bottom. The values of α vary between 1 and -1 where α = 1 represents a nonabsorptive rigid boundary with 100 percent reflection, α = 0 corresponds to a complete absorption with no reflection, and α = -1 characterizes 100 percent reflection from a free surface with an attendant phase reversal (water surface). Recent field investigations have indicated that the average values of α for the reservoir bottom materials measured at several concrete damsites varied over a range from -0.55 to 0.66 (Ghanaat and Redpath 1995). Three of the measured values were negative and the largest (0.66) was much less than 1—the value corresponding to a rigid boundary. The results also showed that some sites had thick layers of soft and muddy sediments with propagation velocities less than that of water, thus leading to negative values of α, a situation never before considered analytically.

b. All hydrodynamic pressure terms (i.e., added mass, added damping, and added force) are affected by the reservoir bottom absorption. Previous studies (Hall and Chopra 1980; Fenves and Chopra 1984b; Fok and Chopra 1985) indicate that the reservoir bottom absorption increases the effective damping, hence reduces the dam response to earthquake loading. The reduction of dam response due to reservoir bottom absorption, however, is much larger for the response to vertical ground motion than to horizontal. Considering that the dam responses due to the vertical and horizontal components of the ground motion are not usually in phase, the effect of reservoir bottom absorption on the total response of the dam is less than that for the vertical ground motion.

2-23. Foundation-Structure Interaction

Foundation-structure interaction introduces flexibility at the base of the structure and provides additional damping mechanisms through material damping and radiation. The interaction with the flexible foundation affects the earthquake response of the structure by lengthening the period of vibration and increasing the effective damping of the system. The increase in the damping arises from the energy radiation and material damping in the foundation region. However, interaction with the flexible foundation also tends to reduce the structural damping that the structure would have had in the case of a rigid foundation (Novak and El Hifnawy 1983). For lightly damped hydraulic structures (less than 10 percent damping) the reduction in structural damping is usually more than compensated for by the added damping of the flexible foundation. Such interaction effects introduce frequency-dependent interacting forces at the structure-foundation interface, which are represented by the dynamic stiffness (or impedance) matrix for the foundation rock region, as described previously.

2-24. Rock Foundations

a. Massless finite element foundation model.

(1) The effects of dam-foundation interaction can most simply be represented by including, in the finite element idealization, foundation rock or soil region above a rigid horizontal boundary. The response to the earthquake excitation applied at the rigid base (bedrock) is then computed by the standard procedures. Such an approach, however, can lead to enormous foundation models where similar materials extend to large depths and there is no obvious "rigid" boundary to select as a fixed base. Although the size of foundation model can be reduced by employing viscous or transmitting boundaries to absorb the wave energy radiating away from the dam (Lysmer and Kuhlemeyer 1969), such viscous boundaries are not standard features of the general-purpose structural analysis programs.

(2) These difficulties can be overcome by employing a simplified massless foundation model, in which only the flexibility of the foundation rock is considered while its inertia and damping effects are neglected. The size of a massless foundation model need not be very large so long as it provides a reasonable estimate of the flexibility of the foundation rock region. A foundation model that extends one dam height in the upstream, downstream, and downward directions usually suffices in most cases. Unlike the homogeneous viscoelastic half plane model described previously, this approach permits different rock properties to be assigned to different elements, so that the variation of rock characteristics with depth can be considered. The massless

foundation model also permits the earthquake motions recorded on the ground surface to be applied directly at the fixed boundaries of the foundation model. This is because in the absence of wave propagation, the motions of the fixed boundaries are transmitted to the base of the dam without any changes.

b. Viscoelastic foundation rock model.

(1) The stiffness and damping characteristics of foundation-structure interaction in a viscoelastic half-plane (2-D) or half space (3-D) model are described by the impedance function. Mathematically, an impedance function is a matrix that relates the forces (i.e., shear, thrust, and moment) at the base of the structure to the displacements and rotations of the foundation relative to the free field. The terms in an impedance function are complex and frequency dependent with the real component representing the stiffness and inertia of the foundation and the imaginary component characterizing the radiation and material damping.

(2) Viscoelastic half plane model. For sites where essentially similar rocks extend to large depths, the foundation rock for 2D analyses may be idealized as a viscoelastic half plane. In other situations where soft or fractured rock overlies harder rock at shallow depth, a finite element idealization (*a* above) that permits for material nonhomogeneity and structural embedment would be more appropriate. In viscoelastic half plane idealization, foundation-structure interaction is represented by a complex valued impedance or dynamic stiffness matrix ($S_f(\omega)$ in Equation 2-2). Assuming that the structure is supported on a horizontal ground surface with homogeneous material properties, the dynamic stiffness matrix $S_f(\omega)$ is evaluated using the approach by Dasgupta and Chopra (1979) or other approaches that use boundary element and Green's functions to analyze the problem (Wolf and Darbre 1984; Alarcon, Dominguez, and Del Cano 1980).

(3) Viscoelastic half space model. The foundation rock for 3-D analyses of concrete hydraulic structures supported on essentially similar rocks with homogeneous material properties may be represented by viscoelastic half space. Employed in the substructure method of analysis, the half space model leads to an impedance matrix for the foundation rock region defined at the structure-foundation interface. A variety of boundary element methods using different Green's functions, finite element techniques in frequency domain using transmitting boundaries, finite element method in time domain, infinite elements, and hybrid methods are available to compute impedance matrices for surface and embedded foundations. Without certain simplifying assumptions, these techniques are computationally demanding and are usually unsuitable for practical applications. One such assumption applied to the analysis of arch dams is to assume that the dam is supported in an infinitely long canyon of arbitrary but uniform cross section and thus break down the problem into a series of two-dimensional boundary problems (Zhang and Chopra 1991). In situations where soft or fractured rock overlies harder rock at shallow depth, a finite element idealization accounting for the material nonhomogeneity should be used.

2-25. Soil and Pile Foundations

a. Discrete spring-mass-damper soil model. Time-history analysis of concrete hydraulic structures including soil-structure interaction effects can be performed in the time domain using a discrete model of the soil. Such a discrete model of the soil consists of frequency-independent springs, dampers, and masses. The simplest model that can be developed for each degree of freedom of a rigid basemat includes a spring and a damper connected to the basemat with a fictitious mass of the soil added to mass of the structure. The frequency-independent coefficients of this SDOF system are obtained by a curve-fitting procedure such that a good agreement between the dynamic stiffness of the SDOF model and that of the actual soil (computed using continuum solutions in *b* below) is achieved. However, the performance of the SDOF system in reproducing the actual response of the soil is limited. A better agreement can generally be achieved by introducing an additional mass connected to the basemat through a damper (Wolf 1988; Figure B-1 of Appendix B). Appendix B provides descriptions and tables of discrete model dimensionless coefficients for a disk supported by a homogeneous half space, an embedded cylinder, an embedded prism, and a strip supported on the surface of a homogeneous half space.

b. *Continuum model*. In the substructure method of analysis in the frequency domain, the soil medium is analyzed separately to obtain the force-displacement relationship of the soil defined at the structure-soil interface. This frequency-dependent impedance function (dynamic stiffness) for the unbounded homogeneous surface foundation can be obtained using continuum solutions. The viscoelastic model described in paragraph 2-24*b*(2) is an example of such procedure, which is equally applicable to analysis of the soil medium.

c. *Finite element model*. The frequency-dependent impedance function (dynamic stiffness) for layered and nonhomogeneous soil medium is obtained using finite element procedures. The finite element analysis for development of impedance functions for 2-D and 3-D problems may be performed using FLUSH (Lysmer et al. 1975) and SASSI (Lysmer et al. 1981) computer programs, respectively. Using these codes, first the equations of motion for the foundation substructure are solved for unit harmonic loads applied at the boundary interface to calculate displacements at corresponding degrees of freedom (dynamic flexibility). The impedance functions are then obtained from the inverse of the dynamic flexibility matrix.